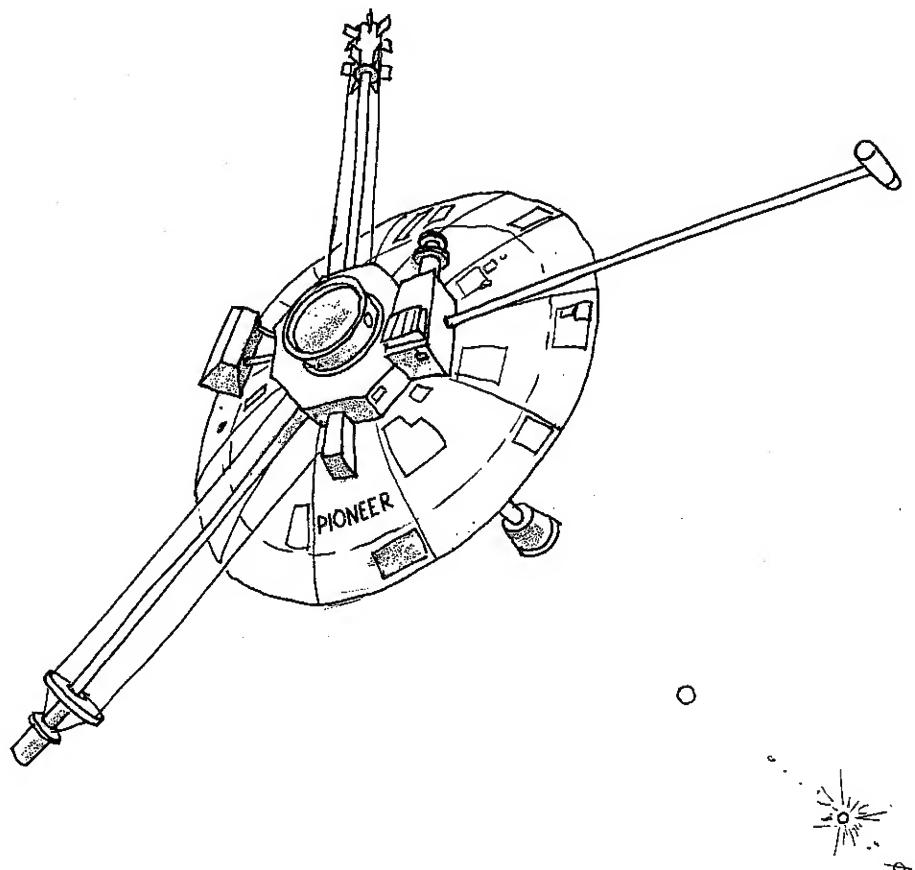
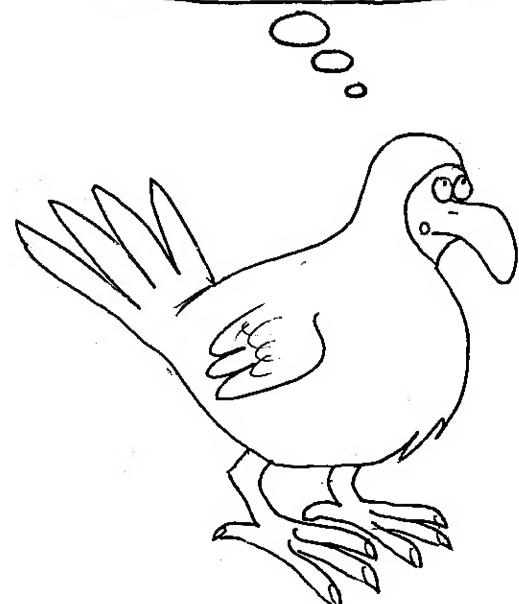


<http://www.savoir-sans-frontieres.com>

# THE TWIN UNIVERSE

Jean-Pierre Petit

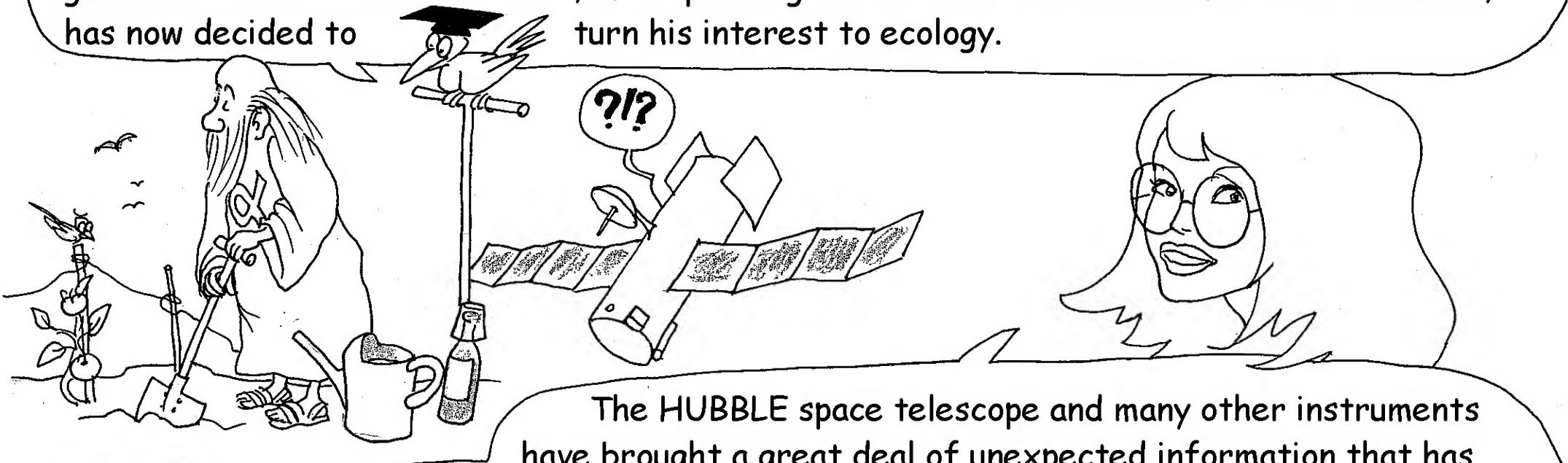
In other words: it's that or  
fiddling around with Newton's law..



Translated by John Murphy

2008

Twenty six years have passed since the author wrote BIG BANG and twenty-two since A THOUSAND MILLION SUNS was published. And what can be said about the twenty-seven years separating us from the BLACK HOLE album? Things have changed enormously since then. Even good old Herbert Reeves himself, after praising the STANDARD MODEL for three decades, has now decided to turn his interest to ecology.

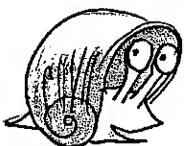


The HUBBLE space telescope and many other instruments have brought a great deal of unexpected information that has plunged astrophysicists into the greatest confusion. The Canadian physicist, Lee Smolin, published a book whose title began "THE TROUBLE WITH PHYSICS..." (in France, Editions Dupond in 2007\*). Perhaps we could also write, in the same vein,

THE TROUBLE WITH ASTROPHYSICS...

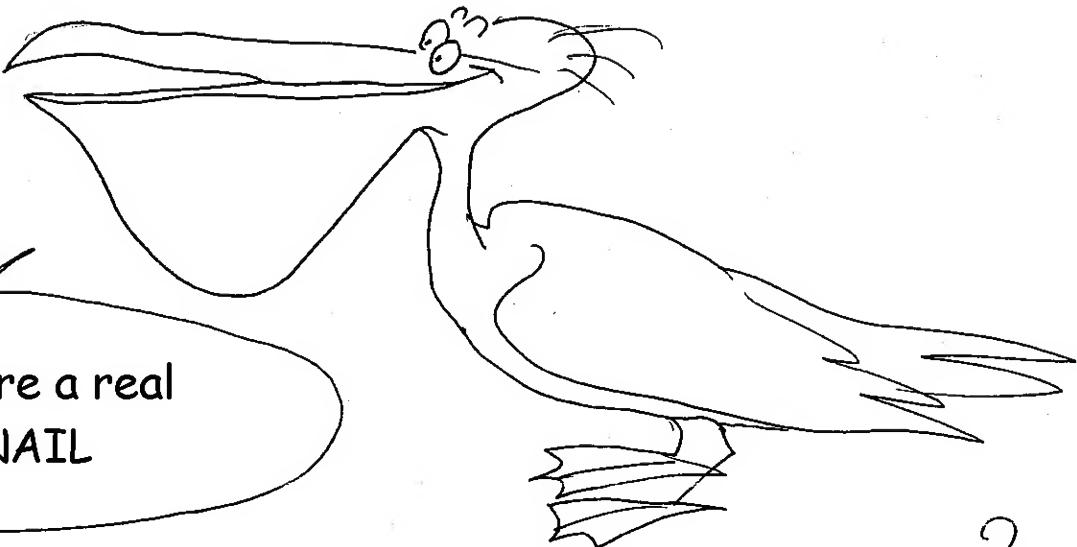
(\*) Original title "The Trouble with Physics - The Rise of String Theory, the Fall of a Science, and What Comes Next"

In any case scientific history shows us that our view of the world has always been evolving. Why should our epoch be any different? Periodically we see a PARADIGM CHANGE. The idea that we have of THINGS and PHENOMENA is profoundly modified. So SPECIAL RELATIVITY and GENERAL RELATIVITY reflect above all a revolution in our conception of the GEOMETRY OF THE UNIVERSE. The growing contradictions, which multiply year after year in astrophysics, that theoreticians try to get round by constantly inventing new words and objects such as DARK MATTER or DARK ENERGY, which we believe can only be resolved with the introduction of a NEW PERCEPTION OF COSMIC GEOMETRY, which we will set out in this album.

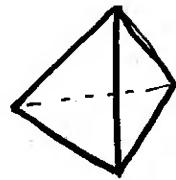


And, as they say:  
"may the best man win".

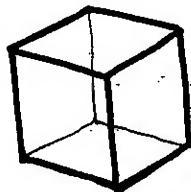
Tiresias, you are a real  
TURBOSNAIL



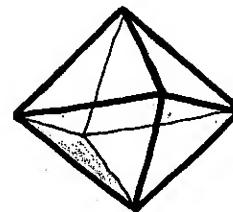
Plato (6th century B.C.) registered four regular polyhedrons (made up of identical faces).



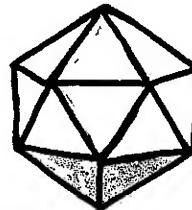
The tetrahedron:  
4 equilateral  
triangles



The cube:  
six square  
faces



The octahedron:  
eight equilateral  
triangles



The icosahedron:  
20 equilateral  
triangles



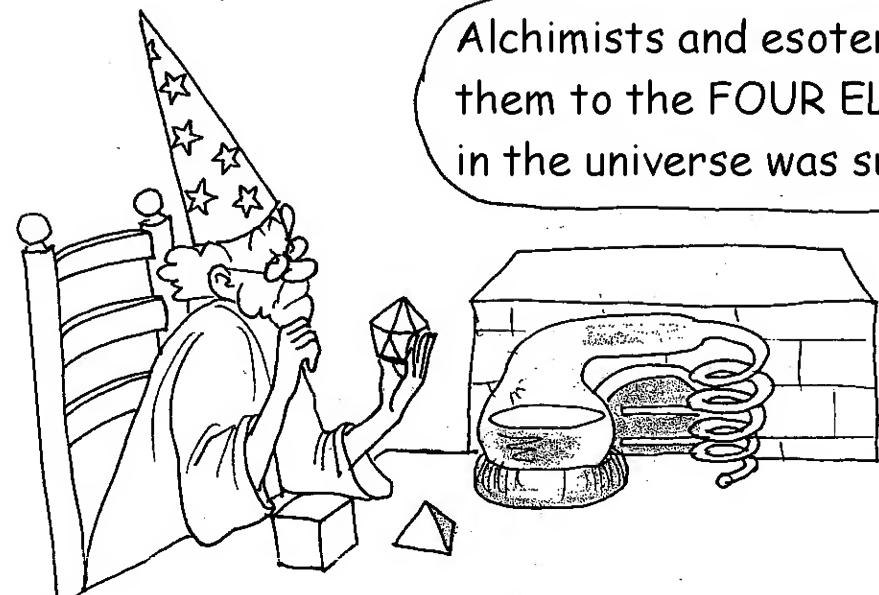
FIRE

EARTH

AIR

WATER

Alchimists and esoterists of all sorts decided to link them to the FOUR ELEMENTS, so that which everything in the universe was supposed to be composed

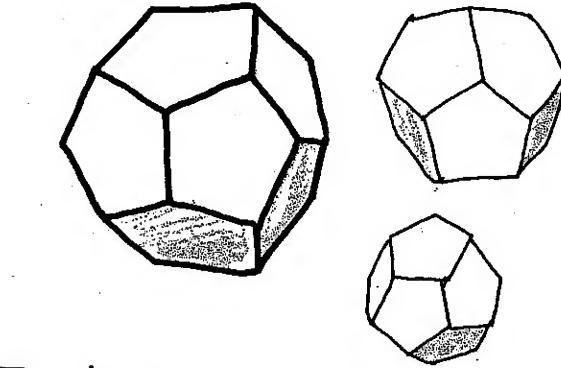


But then there was a catastrophe: a fifth polyhedron was discovered!

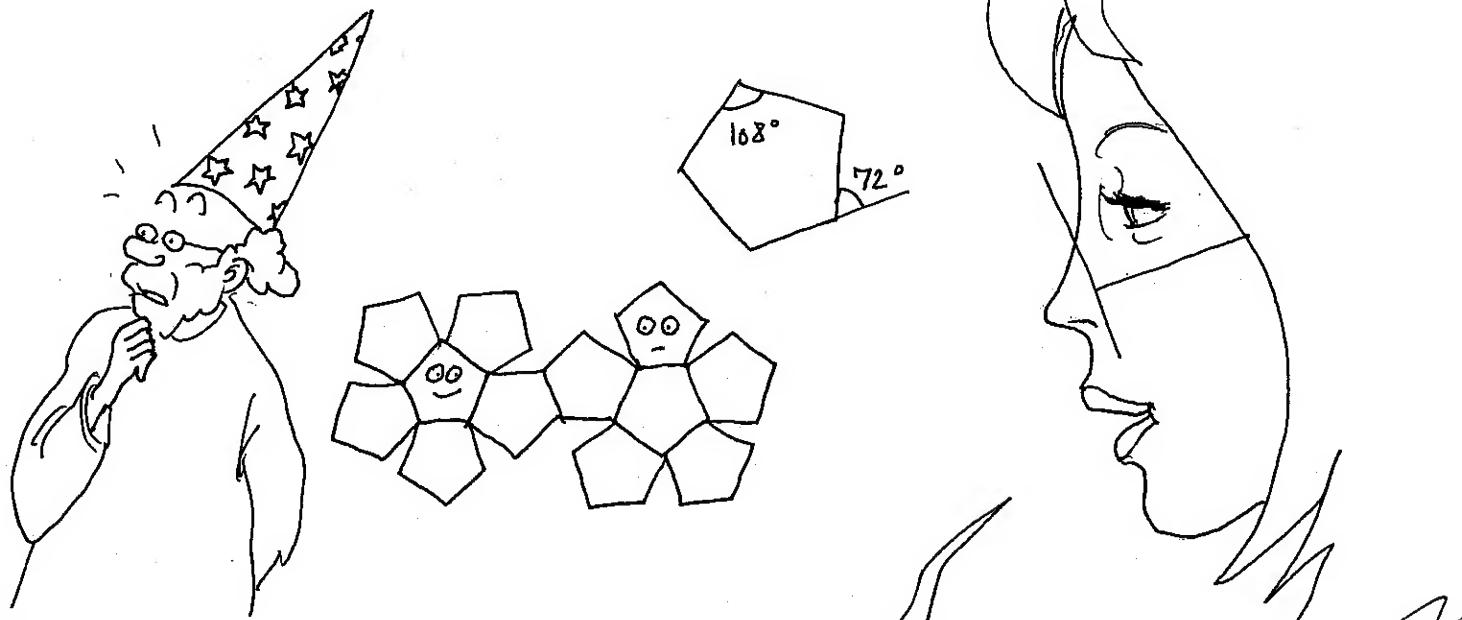


# QUINTESSENCE

## DODECAHEDRON



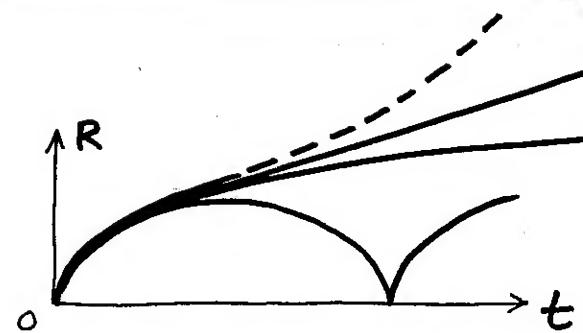
Twelve pentagons



Plato's series of regular polyhedrons continues with the DODECAHEDRON (\*). EDRON is Greek for "face" and DODEKA means "twelve". This polyhedron has twelve pentagonal faces therefore. The "scientists" of antiquity, then those of the Middle Ages, who had brought everything down to the fundamental FOUR ELEMENTS, asked themselves which new ESSENCE this polyhedron referred to. They named it QUINTESSENCE, which means FIFTH ESSENCE.

(\*) we will show that there are exactly five. But see Annex 1

Since 1917 people have believed that the future of the cosmos will bring a more or less marked slowing down of its expansion. However, a few years ago measures made on very distant supernovae showed an incomprehensible ACCELERATION. Astrophysicists invoke a new and remarkable ingredient: DARK ENERGY (originally named "quintessence"!!!)



Have we any idea what this mysterious black energy is?

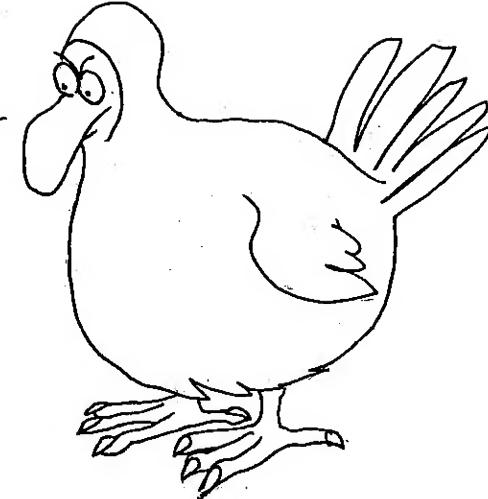
Not a shadow of an idea. All people say is that this component has a REPULSIVE characteristic.

It's like something out of Molière! Once upon a time mercury rose in barometers because nature abhors a vacuum, and everyone knows that sleeping pills work because they have a soporific property. This dark energy completes the menagerie which the mythical DARK MATTER has already joined.

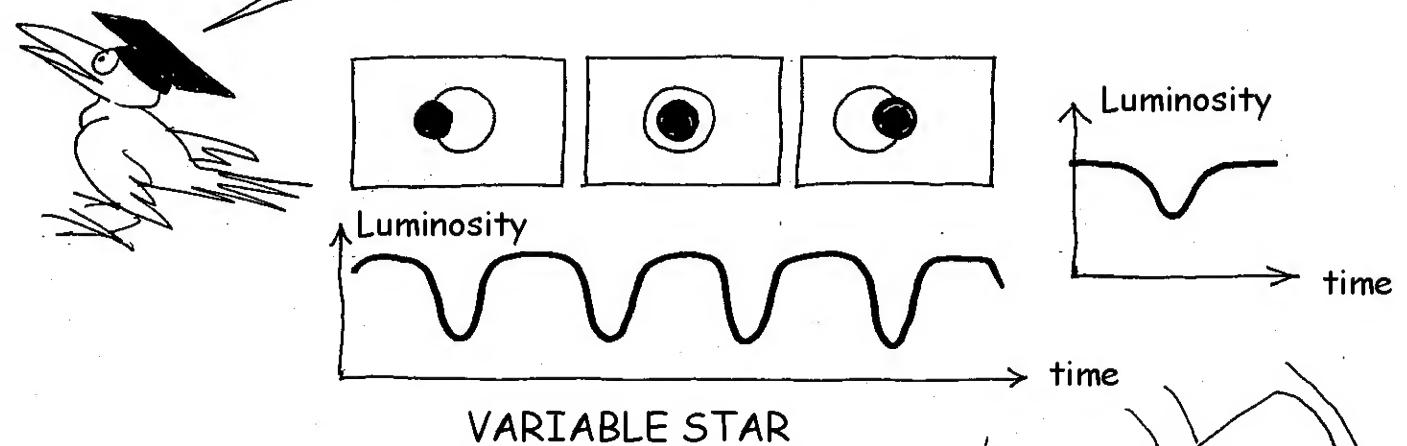


Tiresias,  
stop there!

The existence of  
DARK MATTER is an  
established fact!

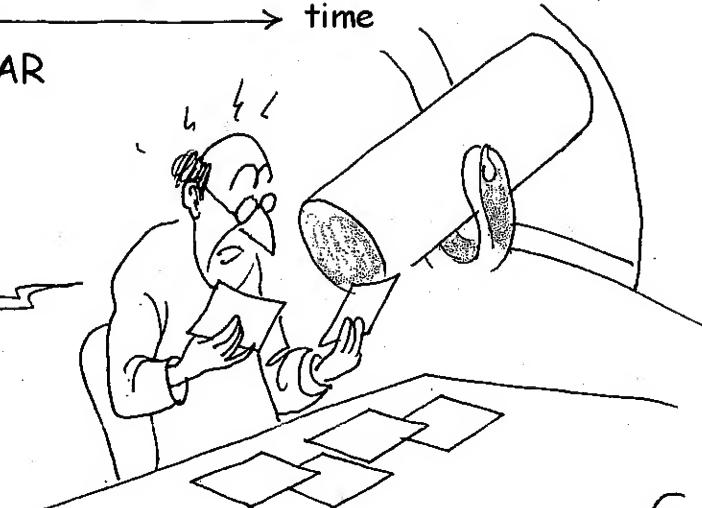


How so an established fact ?! No one has ever managed to show the thing. For twenty years it was thought that it was mini stars or giant Jupiters, MACHOS (\*). They were sought in every direction in the hope that their passage in front of stars would create occultation effects. But every time there was a drop in luminosity it turned out to be because the stars were simple variable stars.



Blast! Just variable stars!!! I've been wasting my time for twenty years!

(\*\*)



(\*\*) Authentic

(\*) Massive Compact Objects: Small objects having mass.

# THE GRAVITATIONAL LENSING EFFECT

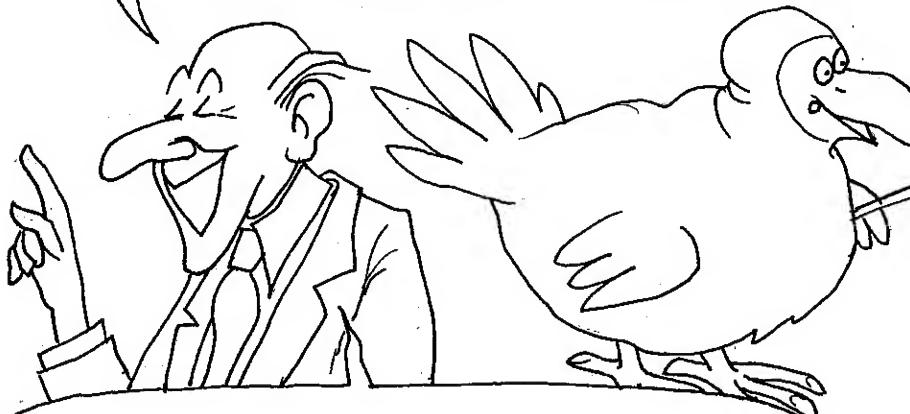
Einstein proposed an identification between mass and curvature in 1917.

From then trajectories of photons became GEODESICS of a hypersurface, which allowed the GRAVITATIONAL LENSING EFFECT to be foreseen and that of GRAVITATIONAL MIRAGES, whose existence was confirmed at the beginning of the nineties.



This observation, gentlemen, is crucial. It shows, with no contestation possible, that DARK MATTER exists. For to obtain such a mirage effect, the mass of the galaxy must be the DOUBLE of what we observe.

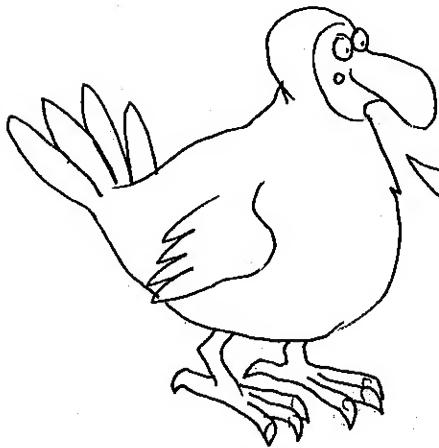
OPTICAL observation has become something secondary, old fashioned. And I shall now give you a second confirmation, absolutely unstoppable.



Therefore, gentlemen, we are entering a new age for astronomy. We can show, because of GRAVITATIONAL EFFECTS, we may never be able to observe them by optical means, whatever their wavelengths be: visible light, ultraviolet, infra-red and even X-rays.

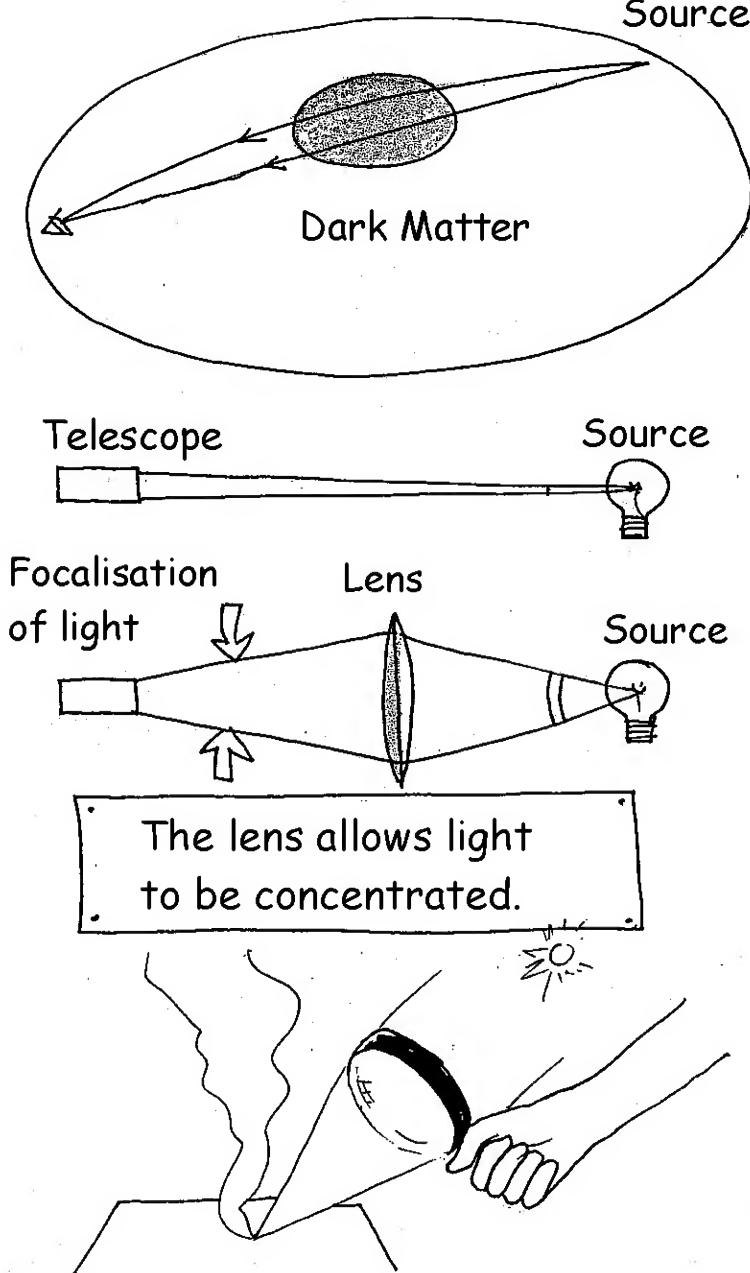
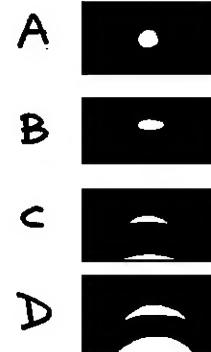
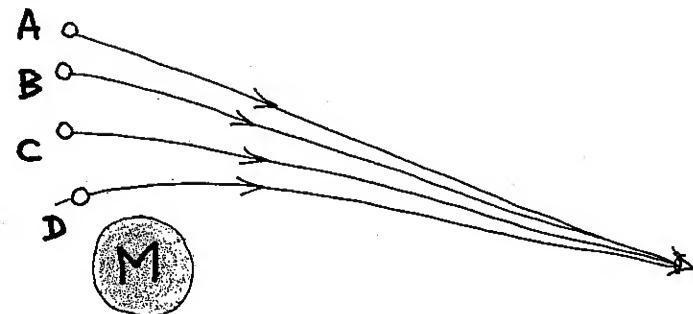


# MICRO-LENSING



Only pessimists say that astrophysics is in a crisis. Our tools have simply developed. So, if light can cross a concentration of dark matter it will be subject to a gravitational lensing effect which will reinforce the source's luminosity just as an optical lens does.

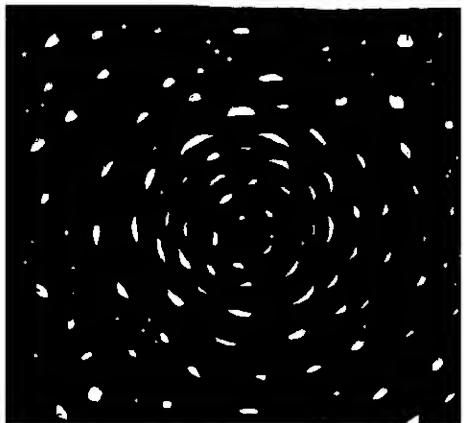
But what is even more interesting is that the gravitational lensing effect deforms the image of galaxies. This can make spheroidal galaxies seem to be elliptic.



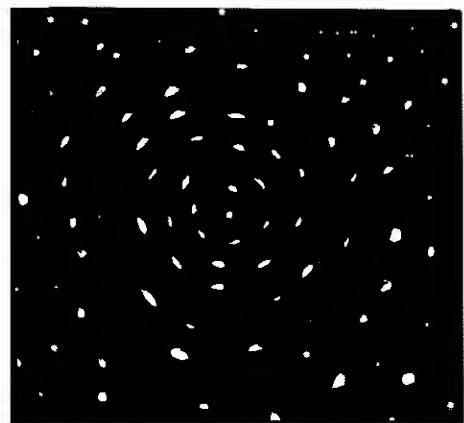
(\*) VISIBLE light, which is an electromagnetic wave, interacts very little with this dark matter, if this does in fact exist, for it emits no radiation and behaves like a completely transparent milieu. Just the gravitational lensing effect remains.



A



B



C

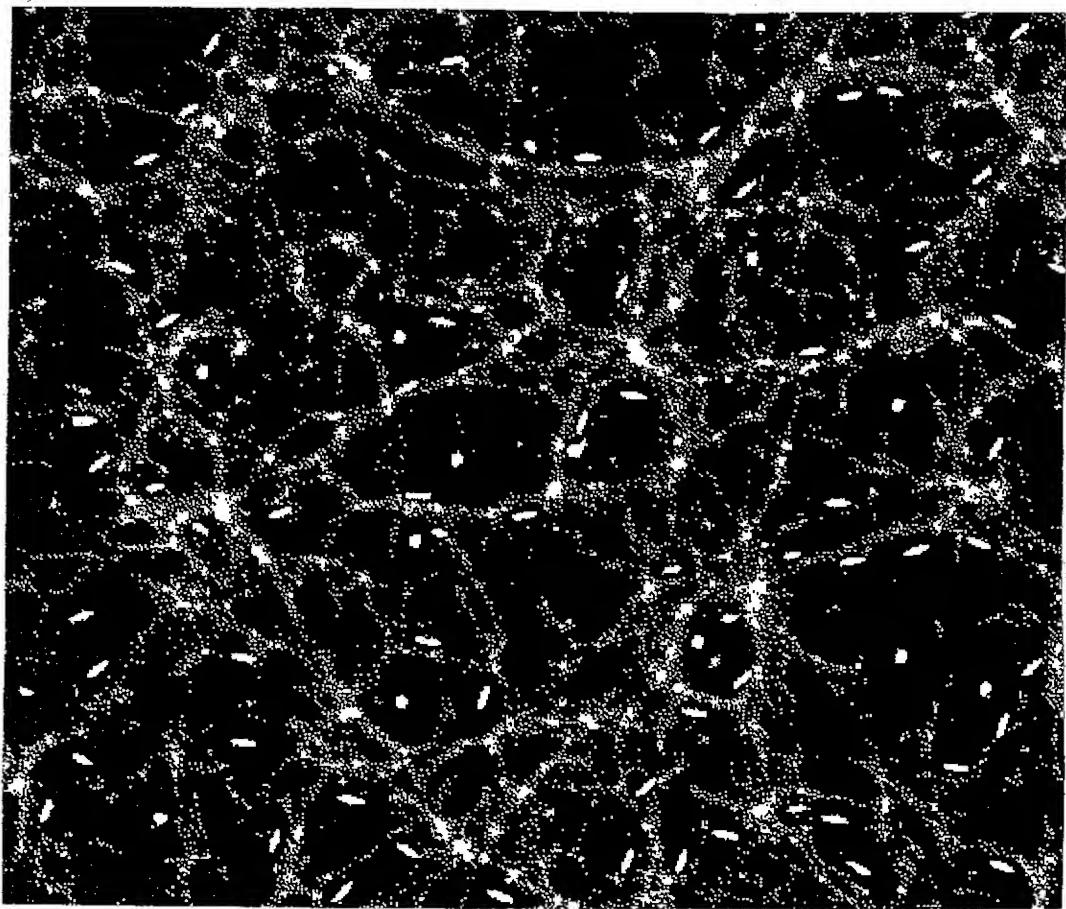
Let us look at a part of the sky sprinkled with distant galaxies - In A, a uniform sky background. In B an invisible object distorts the images of the galaxies by a gravitational lensing effect. Some appear stretched and look like ARCS. The effect is less pronounced in C but remains visible to the naked eye. The study of the image distortion of background galaxies allows the evaluation of the quantity of (dark) matter producing the effect. In the case of **GALAXY CLUSTERS** the mass is often 100 times superior to that measured by counting the visible objects that form the cluster, and whose distance is calculated via their redshift. However the capacity of the human eye is far less than the capacity for analysis and treatment of an image by a computer. From the tiniest (statistical) deformations in background galaxy images they can **MAP** the dark matter in three dimensions (\*)



You mean that using this method we can map something we can't SEE?

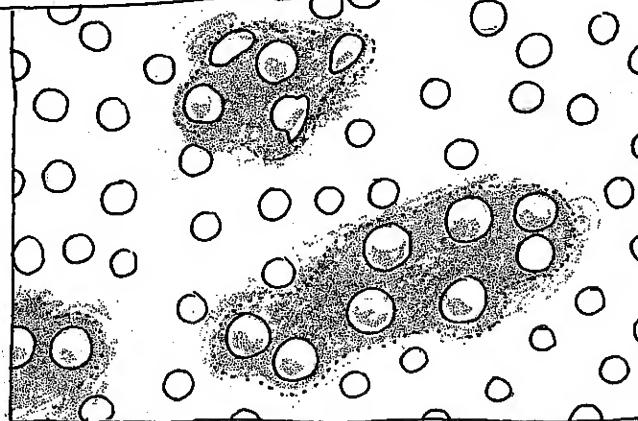
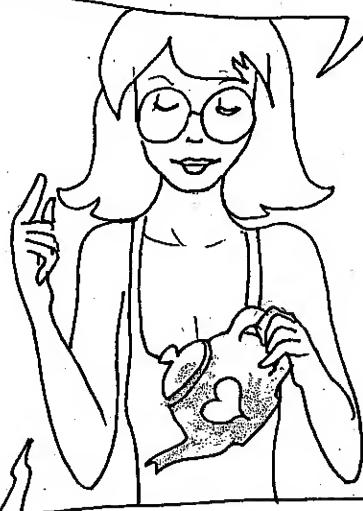
(\*) These techniques were first used at the beginning of the 21st century.

# THE NEW ASTRONOMY



The first map of DARK MATTER published in 2000

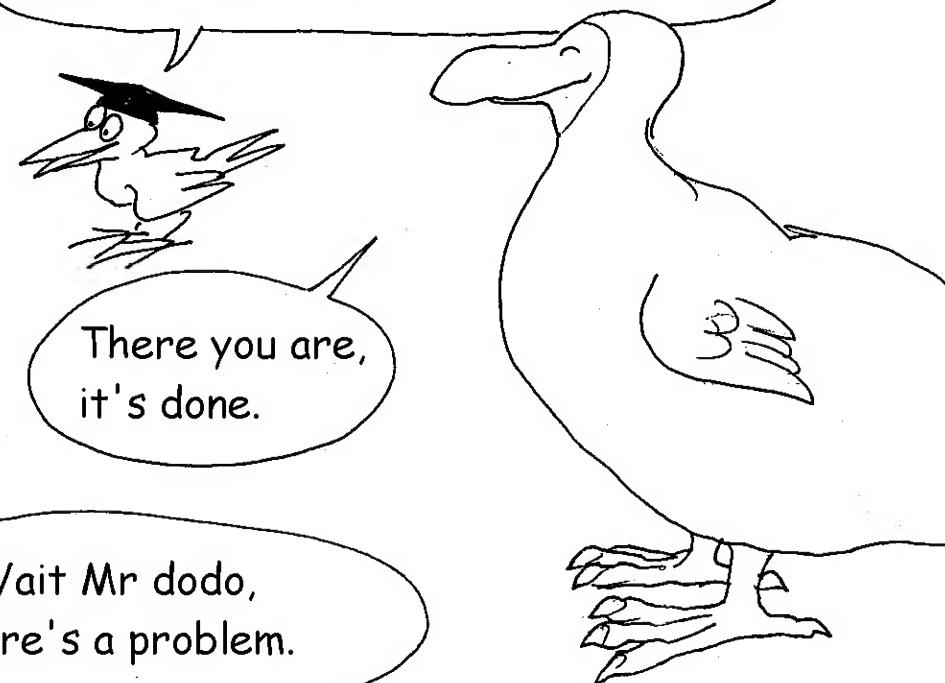
Take an oilskin tablecloth and drop it onto water



Let's suppose that it is covered with white marks on a colored background.



By analysing the distortions of the white marks, through a magnifying effect, a computer would be able to reconstitute the form of the water pools creating the phenomenon, even without seeing the areas of liquid.



There you are,  
it's done.

Wait Mr dodo,  
there's a problem.

Yes Mr Handhic, there is something wrong with this NEW ASTRONOMY

Oh yes, and what is that?

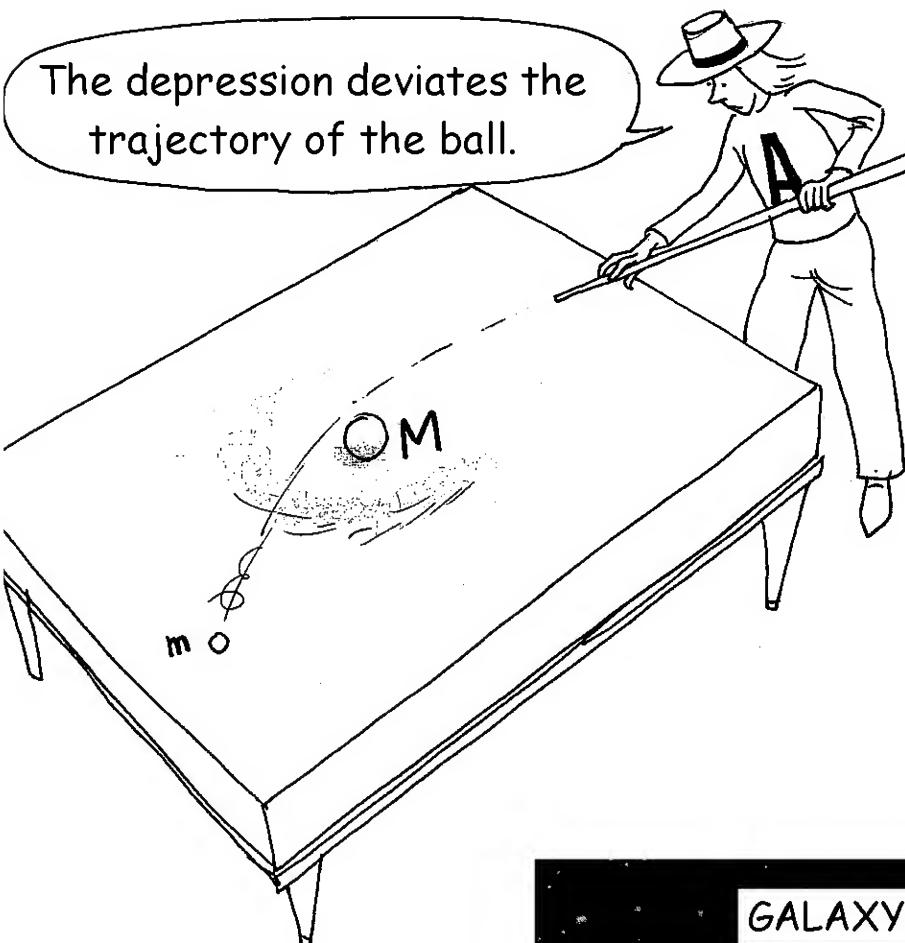
I take this foam mattress which I liken to space. If there is nothing on it and I throw this SOFT BALL it will have a STRAIGHT trajectory: A GEODESIC on a plane.

Photons follow HYPERSURFACE GEODESICS, where Einstein showed we live. Agreed?

If I put a mass  $M$  on the soft ball it will depress the surface, creating a sort of dip. Have I understood the general idea correctly?

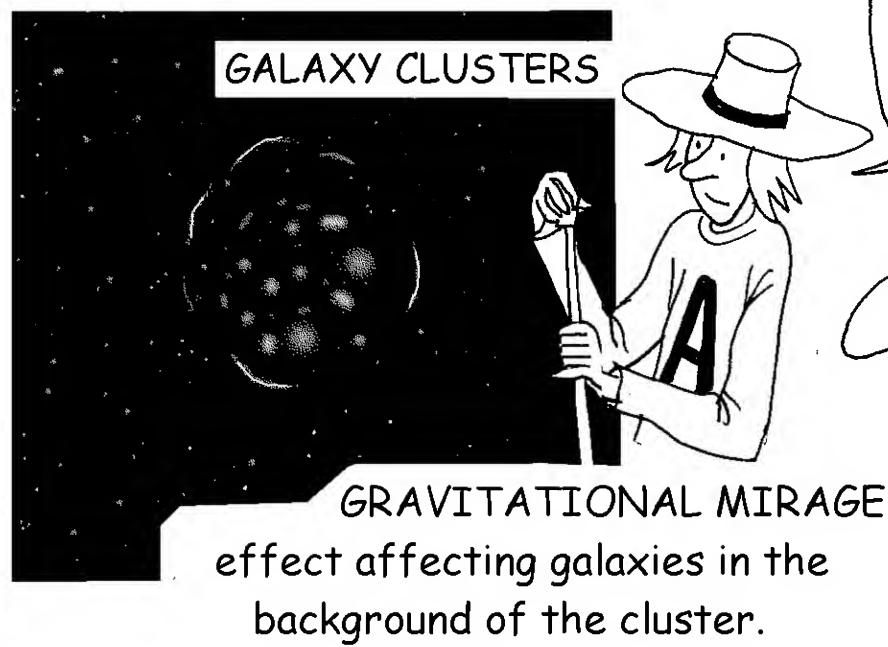
Yes,  
that's right.

The depression deviates the trajectory of the ball.

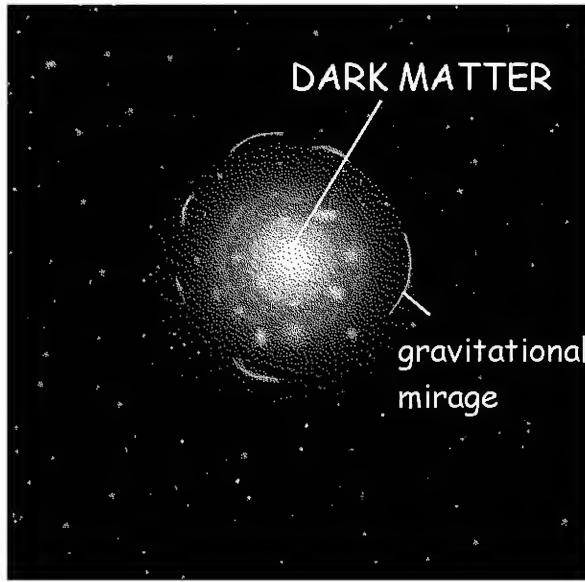


Exactly. Verified in 1919 during a total solar eclipse.

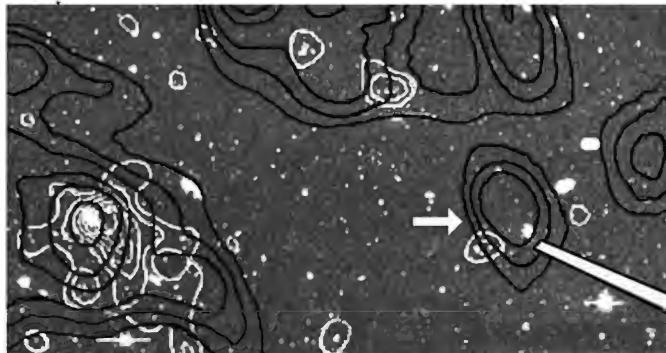
You base your **PROOF OF THE EXISTENCE OF DARK MATTER** on the fact that the gravitational lensing effect observed around certain galaxy clusters is a **HUNDRED TIMES GREATER** than that which would be due to the visible mass, after adding together all the galaxies present in the cluster.



Correct, and so what?



IN 1999 Meillier and Fort localised DARK MATTER CONCENTRATIONS whose mass  $M_{dm}$  was equivalent to a thousand galaxies. But the problem is that on a optical level there was nothing special about the area (\*).



From that you deduce that the mass  $m_{dm}$  of DARK MATTER in the cluster is 100 times greater than the visible mass  $M_v$

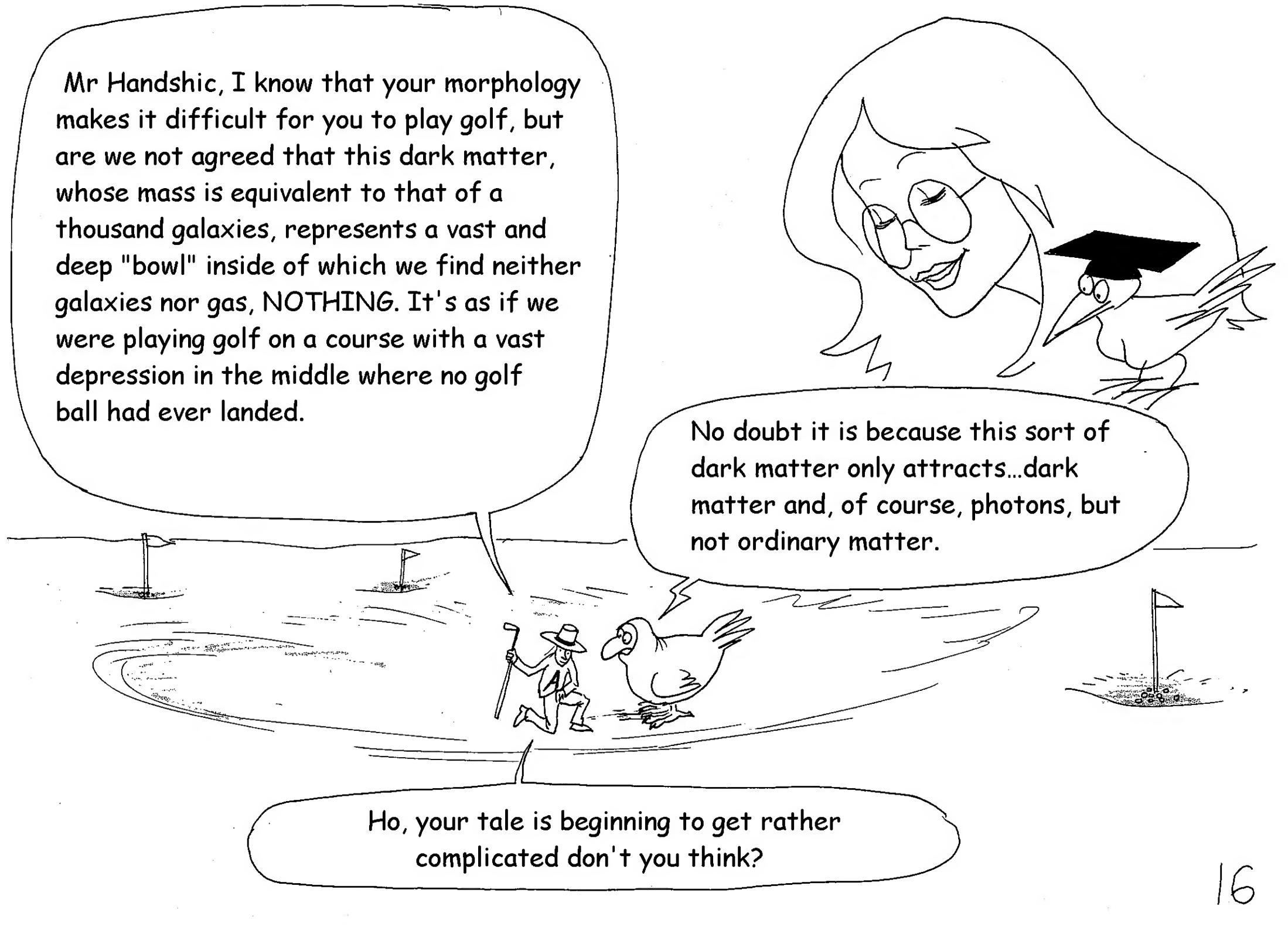
Yes, exactly, what is the problem?

They hunted ordinary matter in every possible frequency: infrared, ultraviolet, but with no result.

It is a DARK CLUSTER entirely composed of dark matter.

(\*) a little way from the Abell 1942 cluster.  
The white arrow indicates the position.

Mr Handhic, I know that your morphology makes it difficult for you to play golf, but are we not agreed that this dark matter, whose mass is equivalent to that of a thousand galaxies, represents a vast and deep "bowl" inside of which we find neither galaxies nor gas, NOTHING. It's as if we were playing golf on a course with a vast depression in the middle where no golf ball had ever landed.

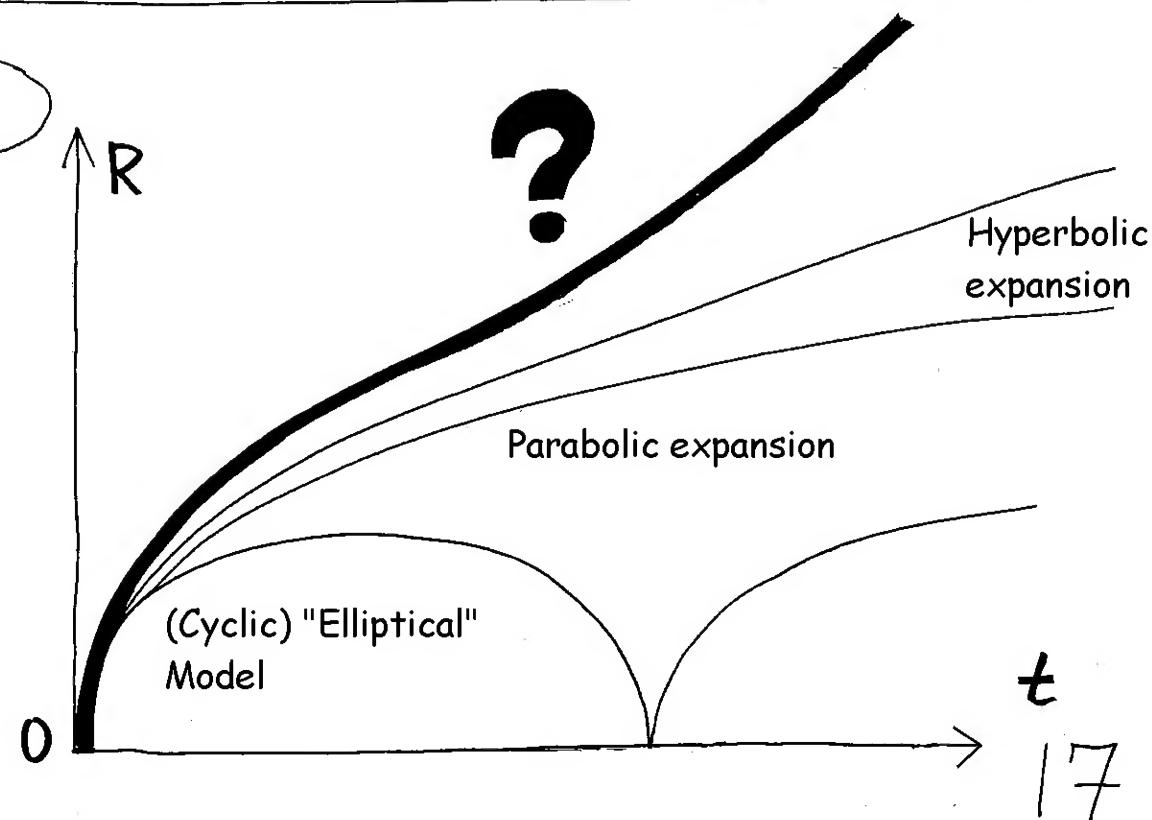


No doubt it is because this sort of dark matter only attracts...dark matter and, of course, photons, but not ordinary matter.

Ho, your tale is beginning to get rather complicated don't you think?

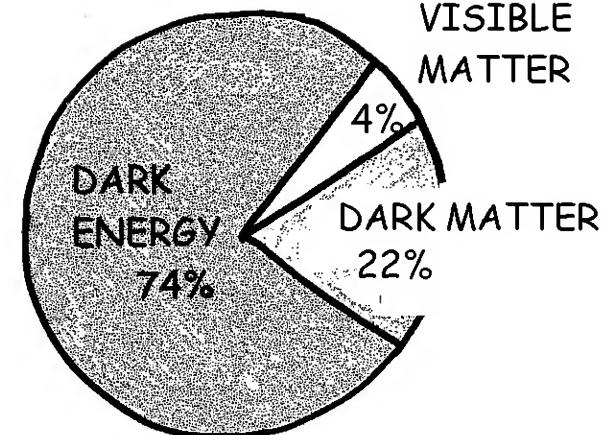
# COSMIC ACCELERATION

As if things weren't bad enough already, observations made at the beginning of the 21st century on extremely distant supernovae confirmed that the cosmic expansion rather than slowing down, as had been thought for three-quarters of a century, was, in fact, accelerating with the passing of time - What could be the mysterious force responsible for such a phenomenon? We didn't know ANYTHING. So a new ingredient was invented to add to a cosmic mix which increasingly resembled the Duck Soup of the Marx Brothers - It was given a name, DARK ENERGY, and endowed with a REPULSIVE FORCE.



To fit the new observational data into the COSMOLOGICAL MODEL astrophysicists came to the conclusion that the Universe was composed of

74% DARK ENERGY,  
22% DARK MATTER  
and 4% VISIBLE MATTER.



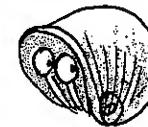
At this point one could ask if there was any point in observing any more, why not simply neglect the miserable 4% that we can see.

Wait, you're forgetting string theory. Thanks to that, one day, it'll all become clear and a THEORY OF EVERYTHING will be proposed.

For the moment it is a THEORY OF NOTHING ...

# PHYSICS AND ASTROPHYSICS SINK INTO AN UNPRECEDENTED HISTORICAL CRISIS

I think it would be interesting to cite a speech given by the president of a university over 20 years ago : "While the string theory has, up until now, produced no interpretation of a phenomenon, has not proposed the least experiment, nor furnished a model of any sort, we note, given the increasing numbers of articles published each year in every country, the extreme vitality of this new discipline (\*)"



A gulf, widening every year, has been created between the spectacular progress of observational and measuring instruments and the capacity of researchers to treat and modelise the data. It is in serious deliquescence. As much as this epoch is in a technological boom, so the fundamental area seems to be in equally great freefall.

(\*) In 2007 the number of articles published was beyond the astronomic figure of a hundred thousand publications and the number of doctoral theses continued to progress.

The PETER SMALL LAW is confirmed day after day.  
It says that the product of the imaginative and creative capacity of a researcher by the power of the computer is a constant.



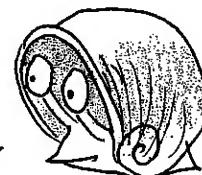
μερδε !

Oh dear, the spiral arms of my galaxy have evaporated again after just one revolution.

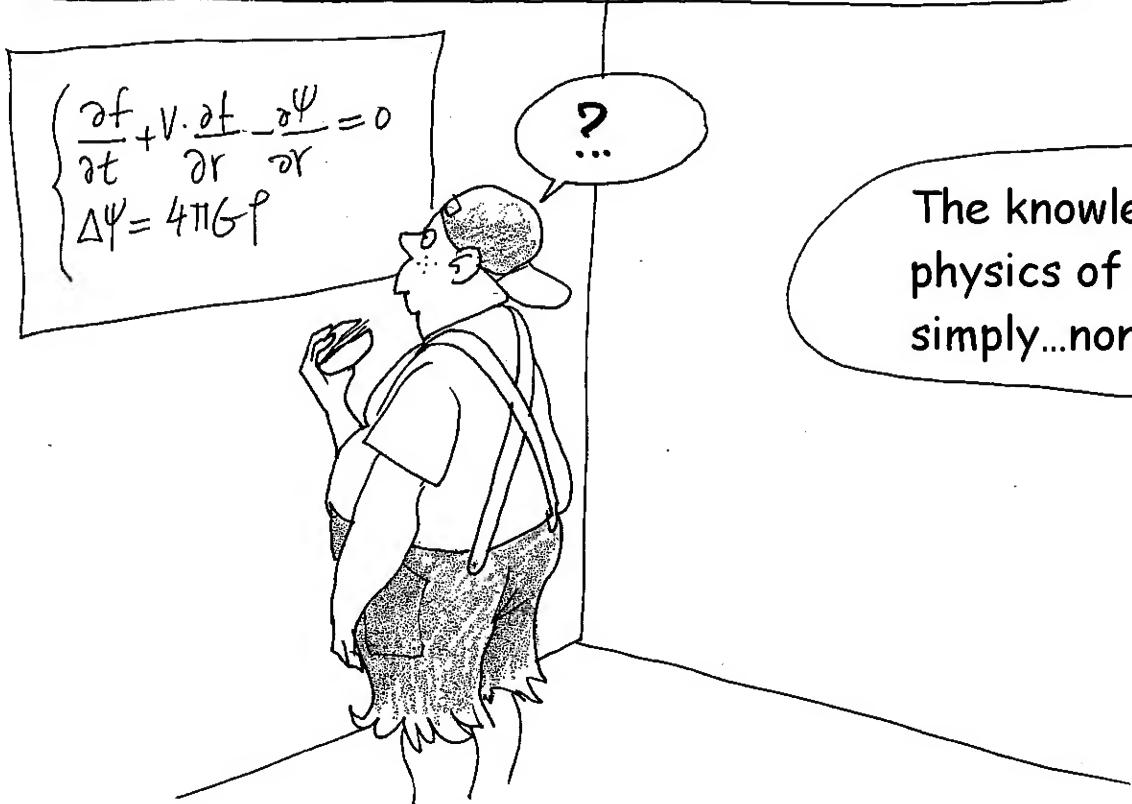


The world's most powerful computer is still no replacement for some well connected neurons.

The key words of this epoch are DIGITAL SIMULATION, a theoretical astrophysicist who has spent his life, unsuccessfully, trying to pierce the mystery of GALAXY DYNAMICS is a researcher who has performed calculations a thousand times, whose theoretical basis is limited to NEWTON'S LAW, who each time changes the parameters hoping that, at last, the miracle will happen.



While we model atoms and the functioning of stars (\*), we have no theoretical model that can describe a galaxy. Our modern "theoreticians" are far from having the knowledge and mental tools of people such as Eddington (\*\*) or Chandrasekhar (\*\*\*)�.



The knowledge of geometry and mathematical physics of the average astrophysicist is quite simply...non-existent.

(\*\*) He calculated the temperature and pressure in the core of stars (1923)  
(\*\*\*) He calculated the limit that bears his name, characterising white dwarves. Nobel Prize in 1983, fifty years after (a record).

(\*) In 1931 a mastery of theoretical calculation allowed the American of Swiss origin FRITZ ZWICKY to predict the phenomenon of supernovae and explain his scenario during a famous conference at CALTECH, long before they were observed and studied..

However, an extremely efficient career-making system has developed thanks to the INTERNET and data bases such as SPIRE which counts up the citations and downloads of articles. This allows ORGANISED GROUPS to inter-evaluate themselves in a completely artificial manner by citing each other - As these groups have also taken over the control systems of scientific journals, by benefitting from the anonymity of the REFEREE system(\*), or have created their own journals, the system is now completely locked into the fields of DOMINANT IDEAS, thus excluding the emergence of any idea, any model that is really innovative. This has allowed the emergence of real SCIENTIFIC IMPOSTURES such as STRING THEORY (which doesn't even exist in the form of an explicit theory).

#### SOME GEMS FROM "THE ELEGANT UNIVERSE" by Brian Greene

4th paragraph cover : A scientific revolution. From the infinitely great to the infinitely small. The unification of all the theories of physics.

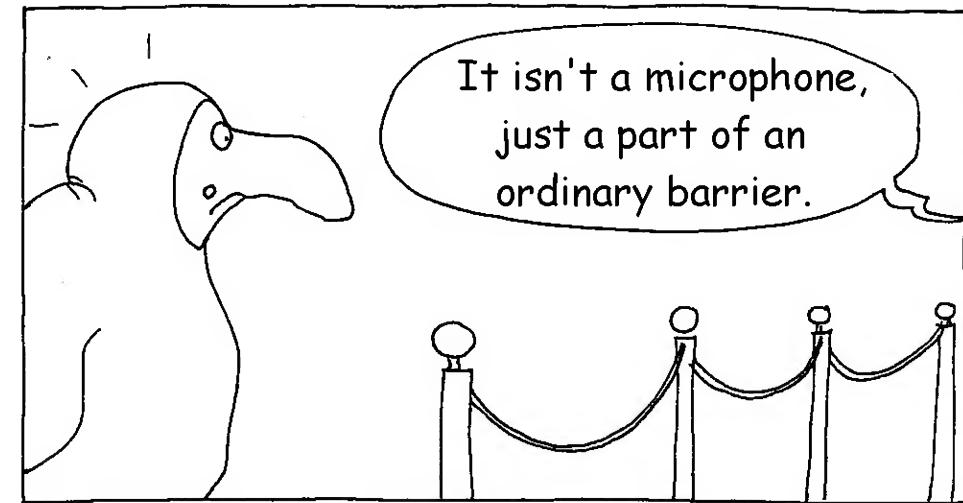
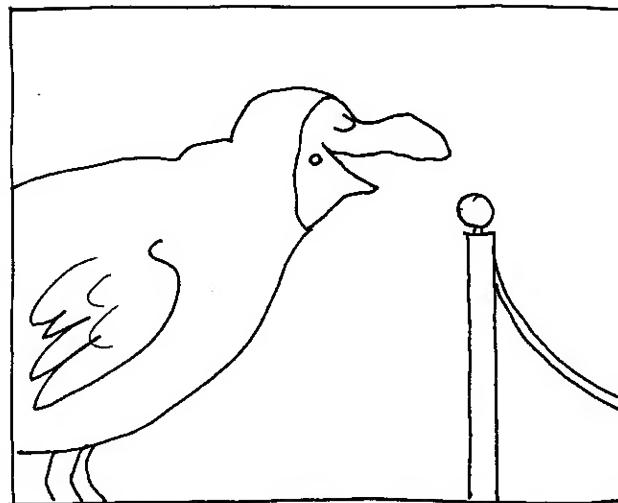
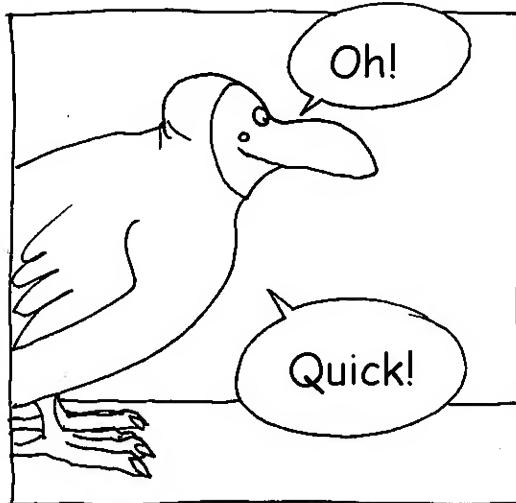
p.189 : We will see that with string theory, while it is the most predictive that physicists have ever known, we are not able to make sufficiently precise predictions to match the experimental data.

p.252 : It is perfectly imaginable that more than one generation of physicists will consecrate their lives to the study of the development of string theory without the least experimental echo.

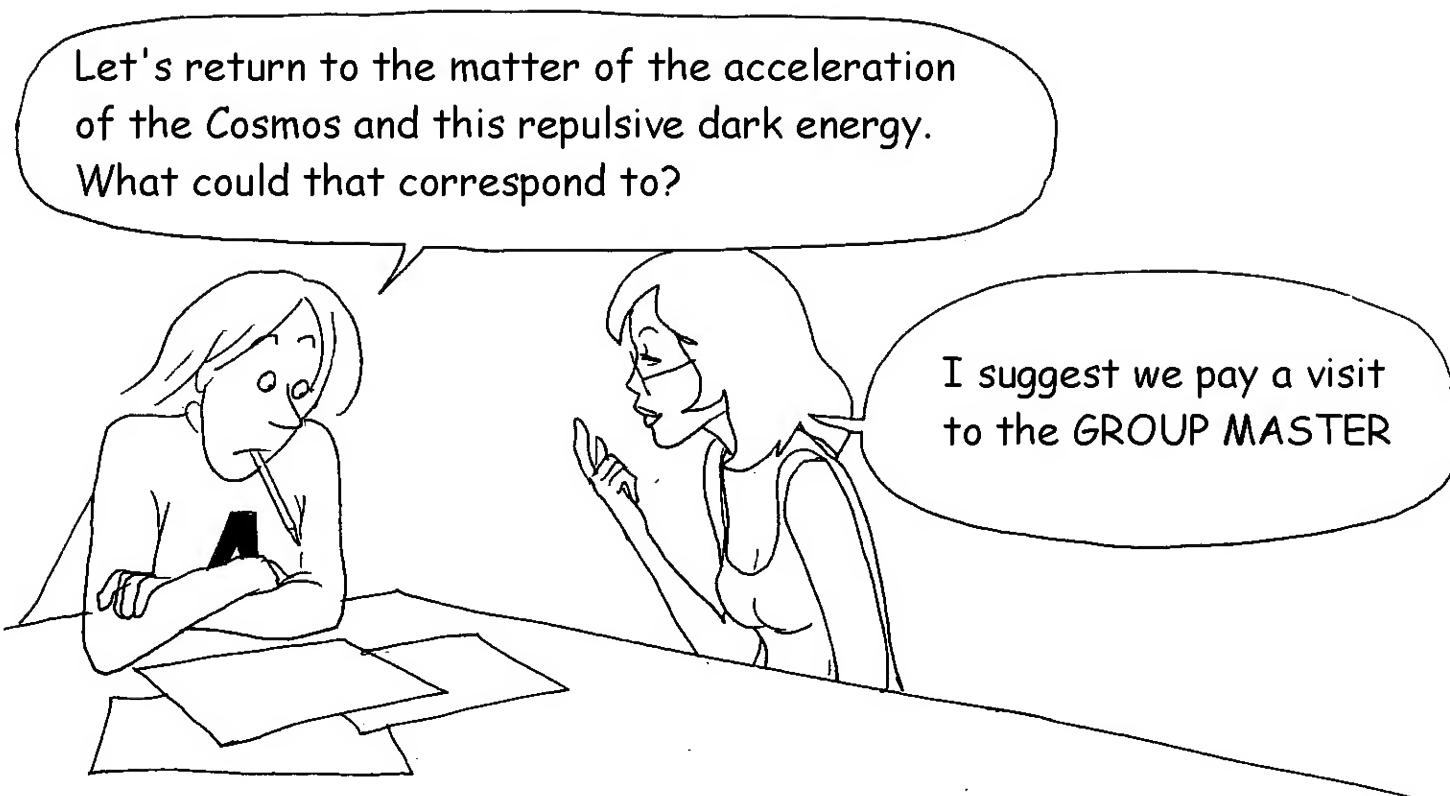
p.300 : Edward Witten (the father of 'cosmic strings' and the mythical "M Theory") is considered the worthy successor to Einstein in the role of the greatest living physicist. Some go further and say that he is the greatest physicist in history (...)

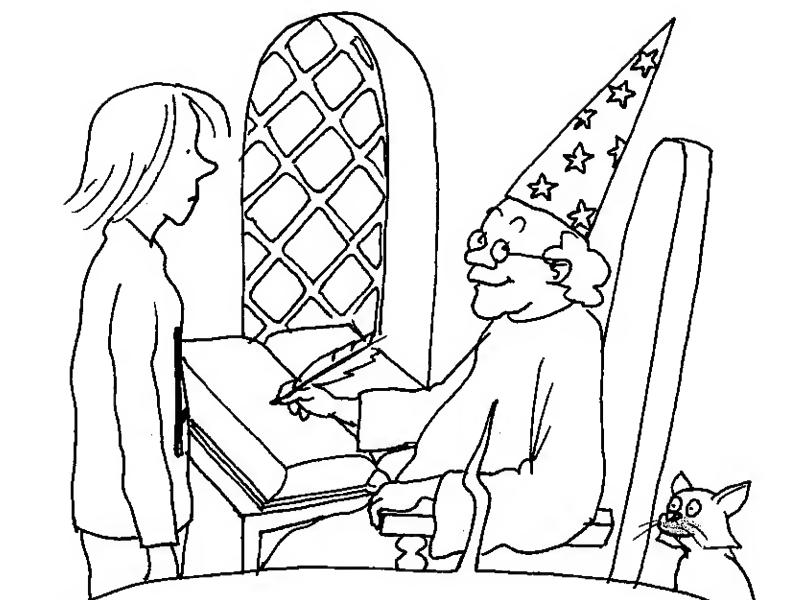


(\*) A system where experts are chosen by the review's office to evaluate a submitted article.

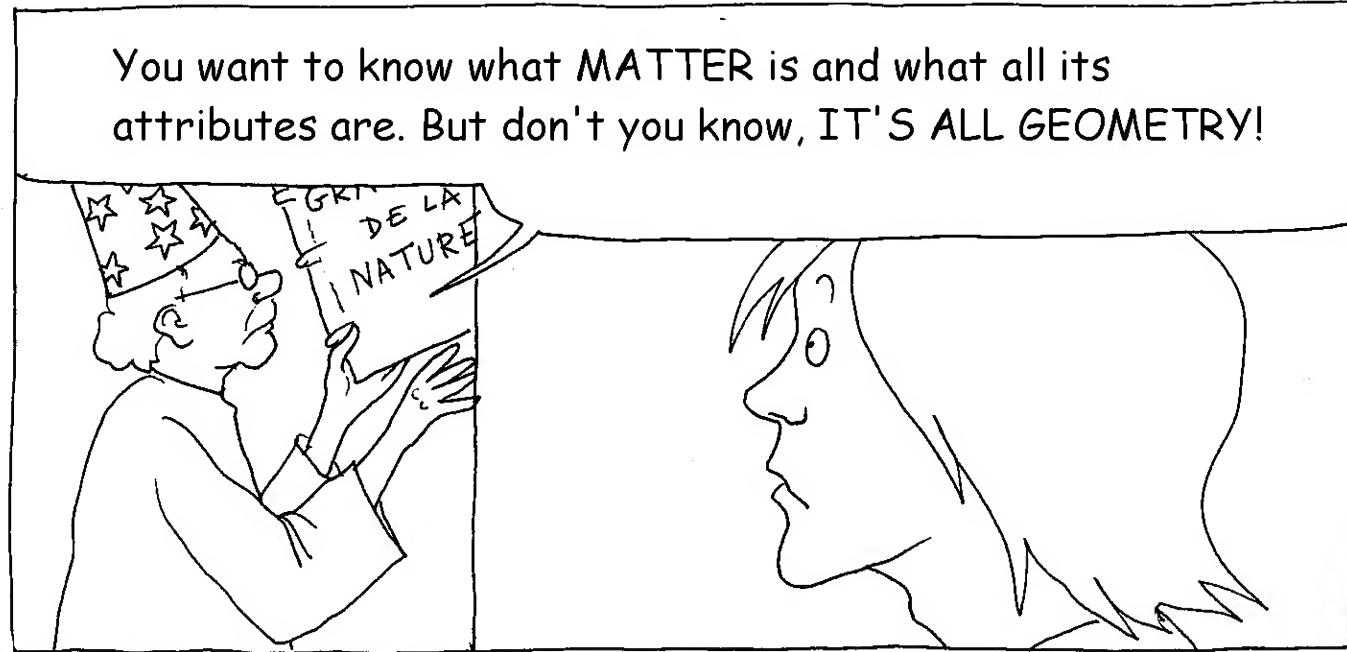


Science today is hyper-mediatised. Unmerited reputations are constructed, where mediocre scientists benefit from an aura born simply out of their vulgarising talents.





Ah, you're back (\*), what brought you here this time?



You want to know what MATTER is and what all its attributes are. But don't you know, IT'S ALL GEOMETRY!

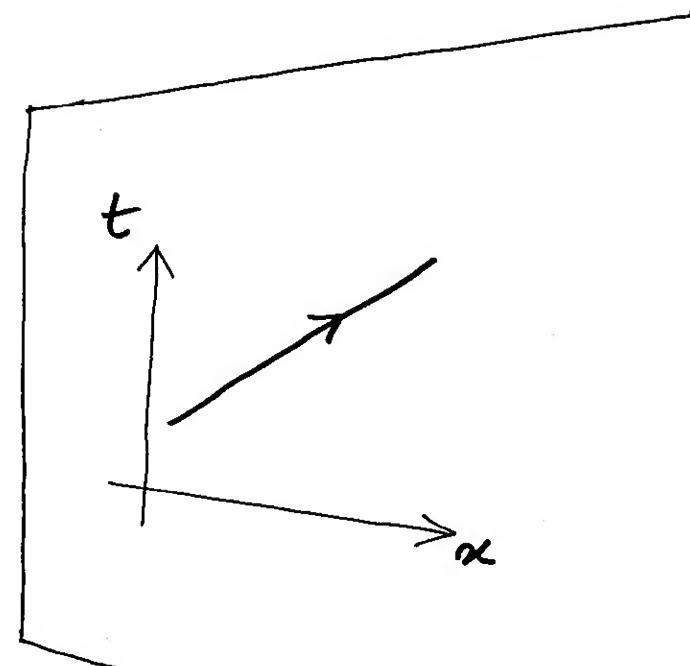
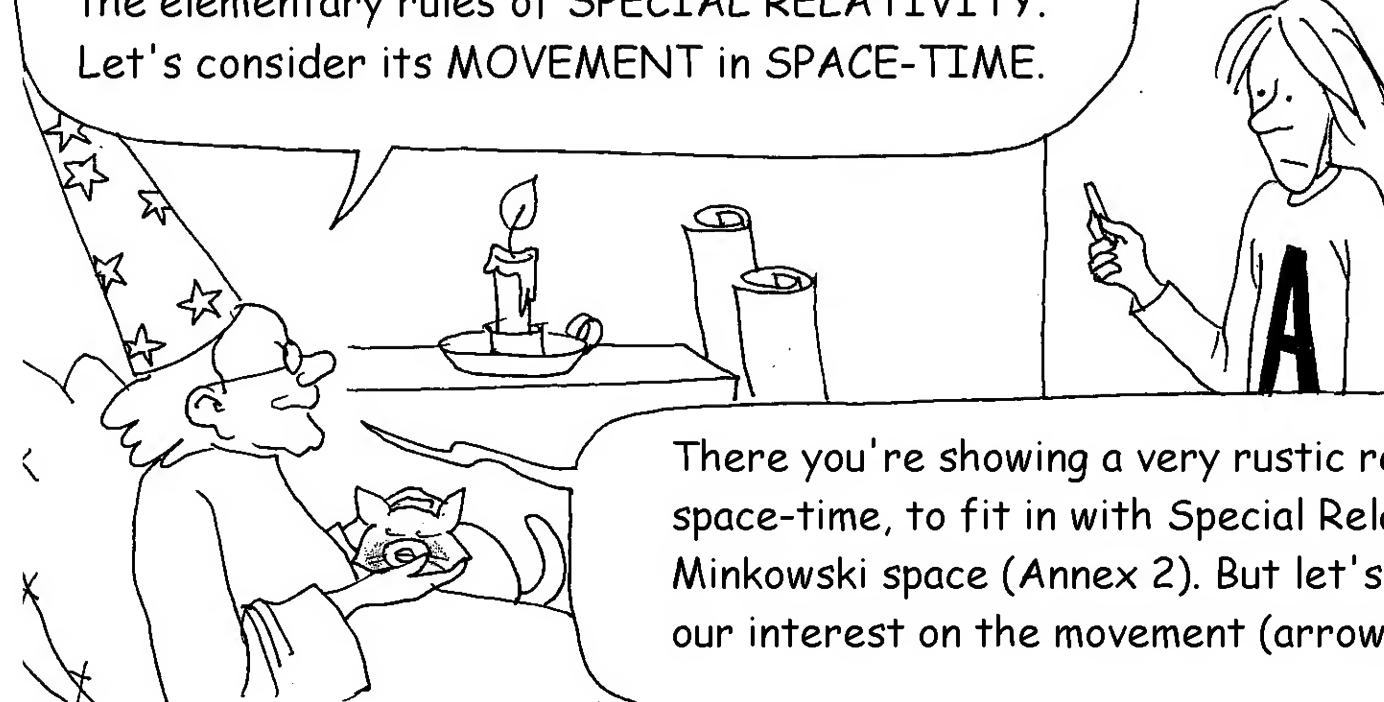


Of course!  
One hundred per cent geometric.

(\*) Reference to an album on GROUPS

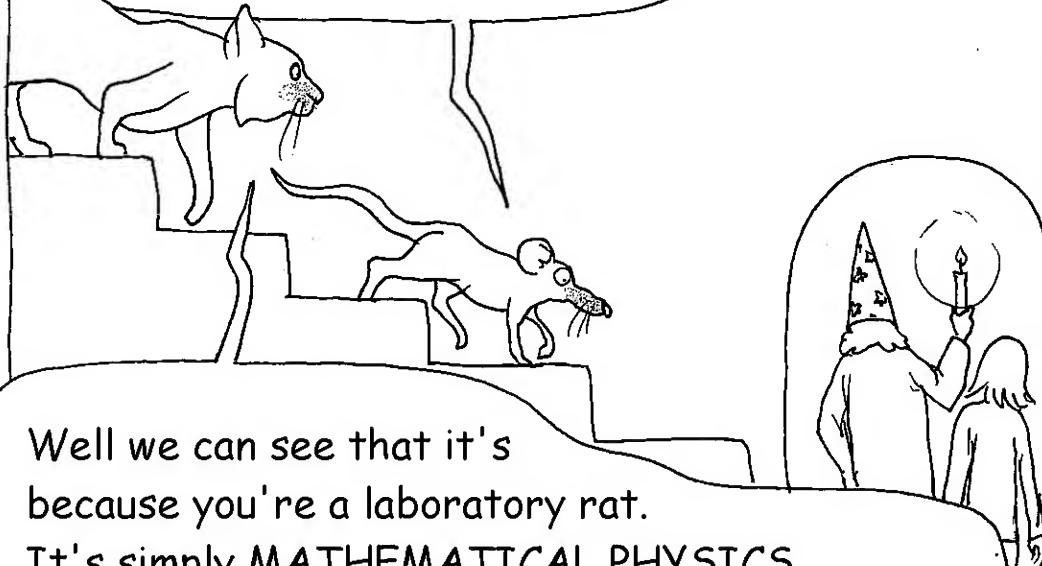
# TELL ME HOW YOU MOVE AND I'LL TELL YOU WHO YOU ARE

Take a material particle and treat it like a RELATIVISTIC MATERIAL POINT, that is, obeying the elementary rules of SPECIAL RELATIVITY. Let's consider its MOVEMENT in SPACE-TIME.



There you're showing a very rustic representation of this movement in space-time, to fit in with Special Relativity it has to be inscribed in a Minkowski space (Annex 2). But let's ignore that detail and concentrate our interest on the movement (arrow).

I feel like I'm descending into  
the catacombs of the Universe.

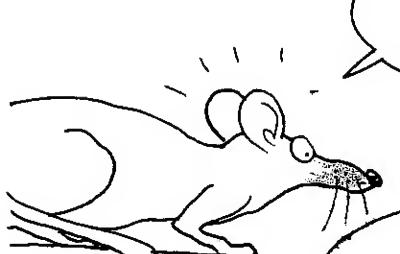


Well we can see that it's  
because you're a laboratory rat.  
It's simply MATHEMATICAL PHYSICS.  
We're going to get to the bottom of things.

Have you ever held in your hands  
a particle of mass  $m$ ? Have you?



I'm wondering if I did the  
right thing in coming here.

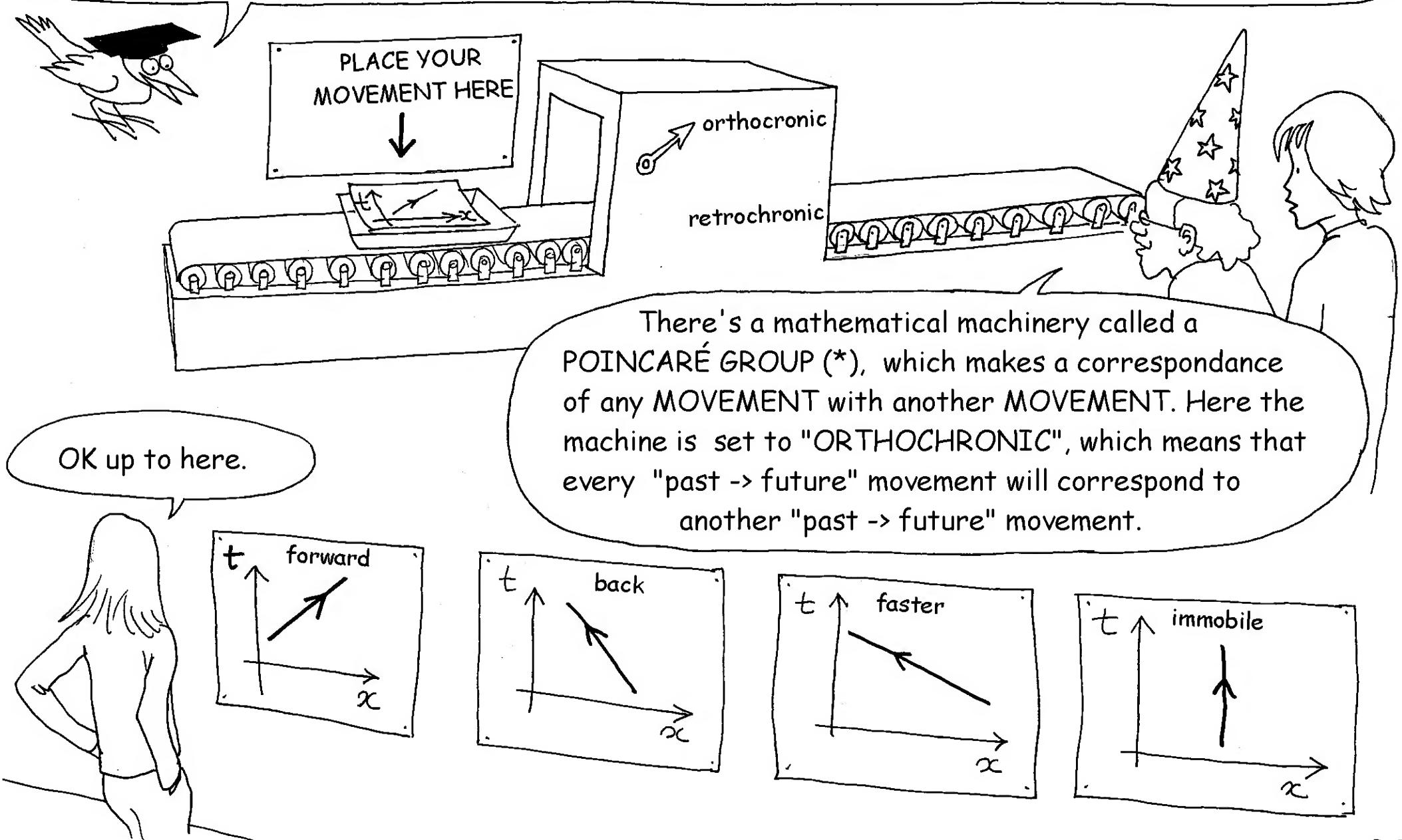


What would you prefer,  
visit a psychiatrist?

Er no, not really. You think you're holding  
something then you no longer really know  
what you're holding anymore.

ABANDON CERTAINTY  
ALL YE WHO ENTER HERE

For readers who have a (little) mathematical baggage, all these things are explained in the Annex.  
For everyone else we'll make do with pictures.



(\*) These secrets are all revealed in the annex.

What's this weird lever with two positions "orthochronic" and "retrochronic"?

It's the key to Pandora's Box.

If you put MOVEMENT in space-time, orthochronic, that is, oriented in the direction past-future, half of the elements of the Poincaré group will transform themselves into another movement with the same temporal orientation, but the remaining half will transform themselves into a "future-past" movement.

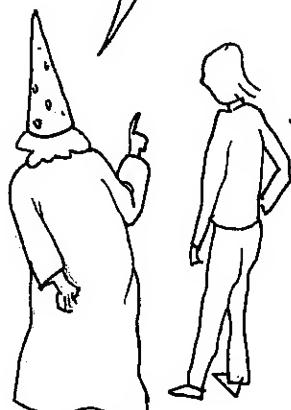
Heavens! Do you mean that there are particles going backwards in time?

The group shows it.

The group and space are closely linked. They mutually confer each other's existence.

Yes but IS the group reality?

That doesn't answer my question = particles going backwards in time, can they exist?



It seems you came because you were asking yourself questions on MATTER. Let's do an experiment then. I consider the past-future movement of a particle of mass  $m$ .

Start the machine except, this time, set it to "retrochronic".

MOVEMENT  
↓

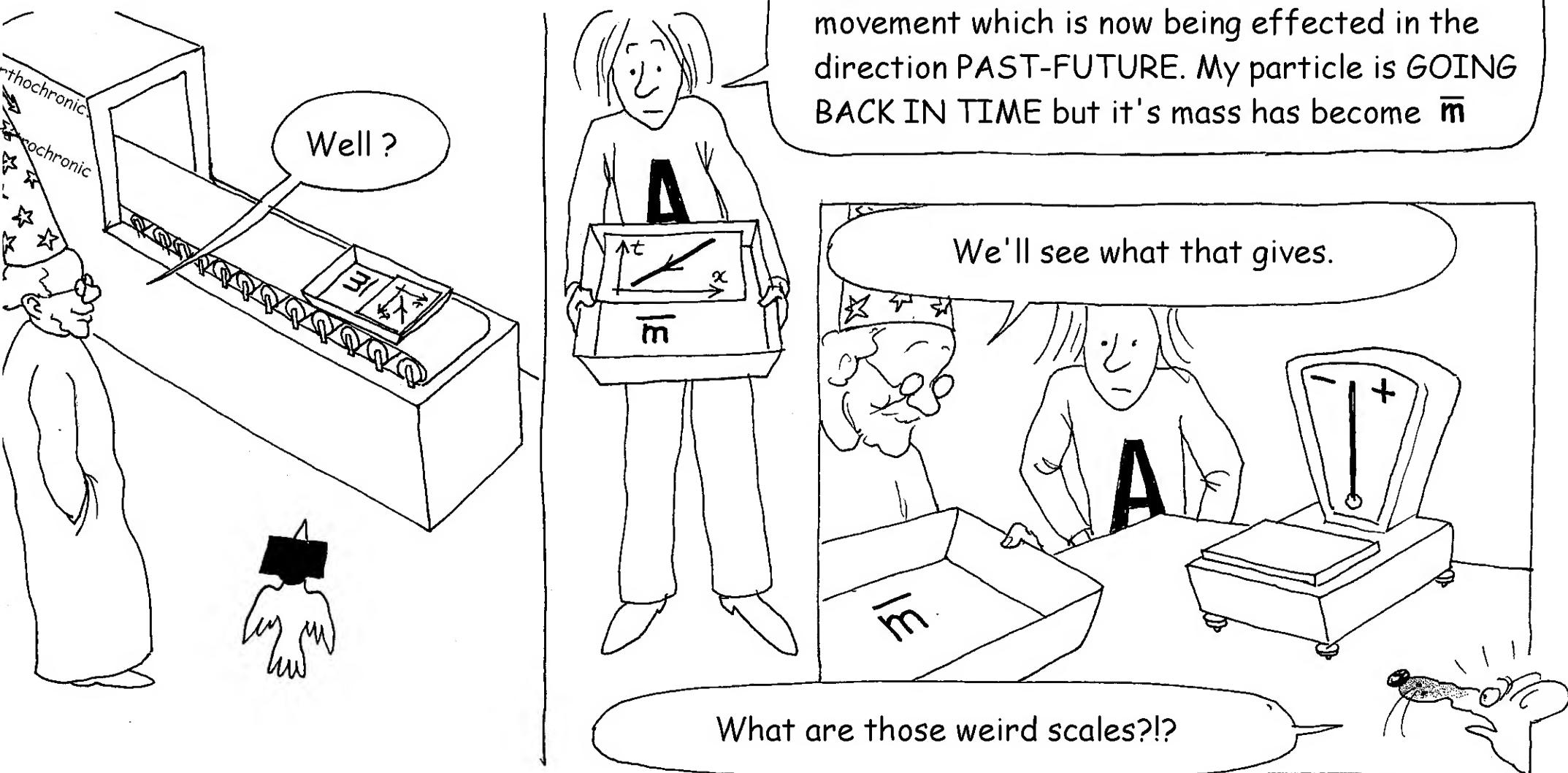
Goodness, I don't like this at all!

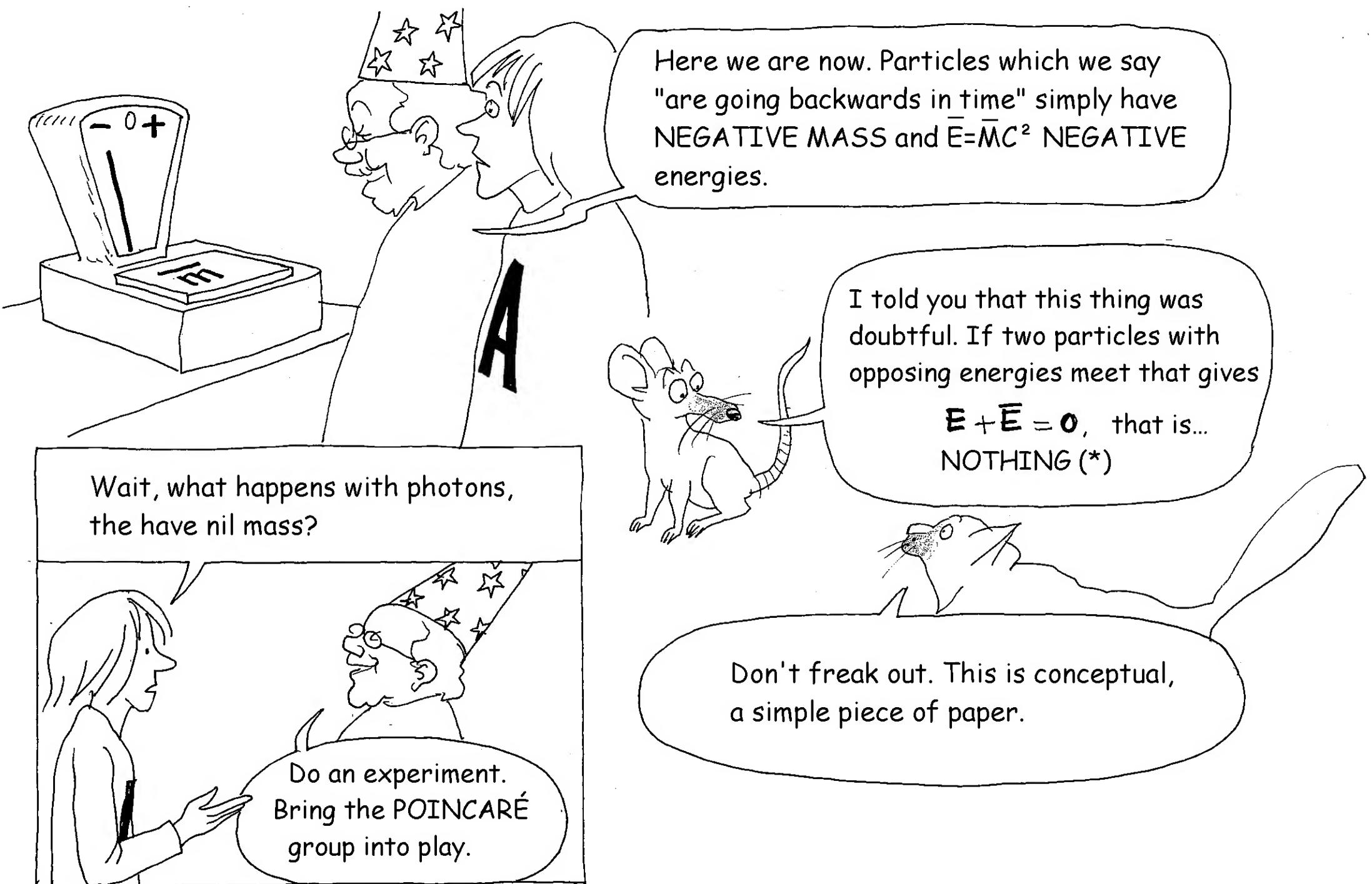
In other words I'm acting on retrochronic elements of the Poincaré Group.

Suspense: The result of this operation is on the next page.

If you're frightened already go and see the superstring people. They won't destabilise you with unforeseen discoveries.

# NEGATIVE MASSES AND ENERGIES





(\*) And not photons as in the so-called MATTER-ANTIMATTER ANNIHILATION where energy is conserved,  
which we really should qualify as DEMATERIALISATION.

OK, let's go with the photon  $\varphi$  whose energy is  $E = \hbar\nu = \frac{\hbar}{\tau}$  where  $\tau$  is the associated wave period.

With the switch still set to "RETROCHRONIC" to turn a "past-future" movement into a "future-past" one.

That seems simple enough.

The photons "going backwards in time" have negative energy

$$\bar{E} = \frac{\hbar}{\bar{\tau}} < 0 \text{ because } \bar{\tau} < 0$$

Your eyes and your measuring instruments aren't capable of capturing the photons with negative energy that are emitted and captured by particles with negative mass  $\bar{m}$

So we can neither see nor observe these negatives masses.

And what about gravity?

Exactly.

Apply :

$$F = \frac{G m m'}{d^2}$$

$m$  and  $m$  mutually attract each other according to **NEWTON**

$\bar{m}$  and  $\bar{m}$  mutually attract each other according to **NEWTON**

$m$  and  $\bar{m}$  mutually repulse each other according to **ANTI-NEWTON**

If I managed to keep a negative mass in a box it would make the box fly because the Earth repulses it.

It would pass through it and, effectively, fly.

But wouldn't it be annihilated with the positive mass particles of the box?

Not even.  
Think about it...

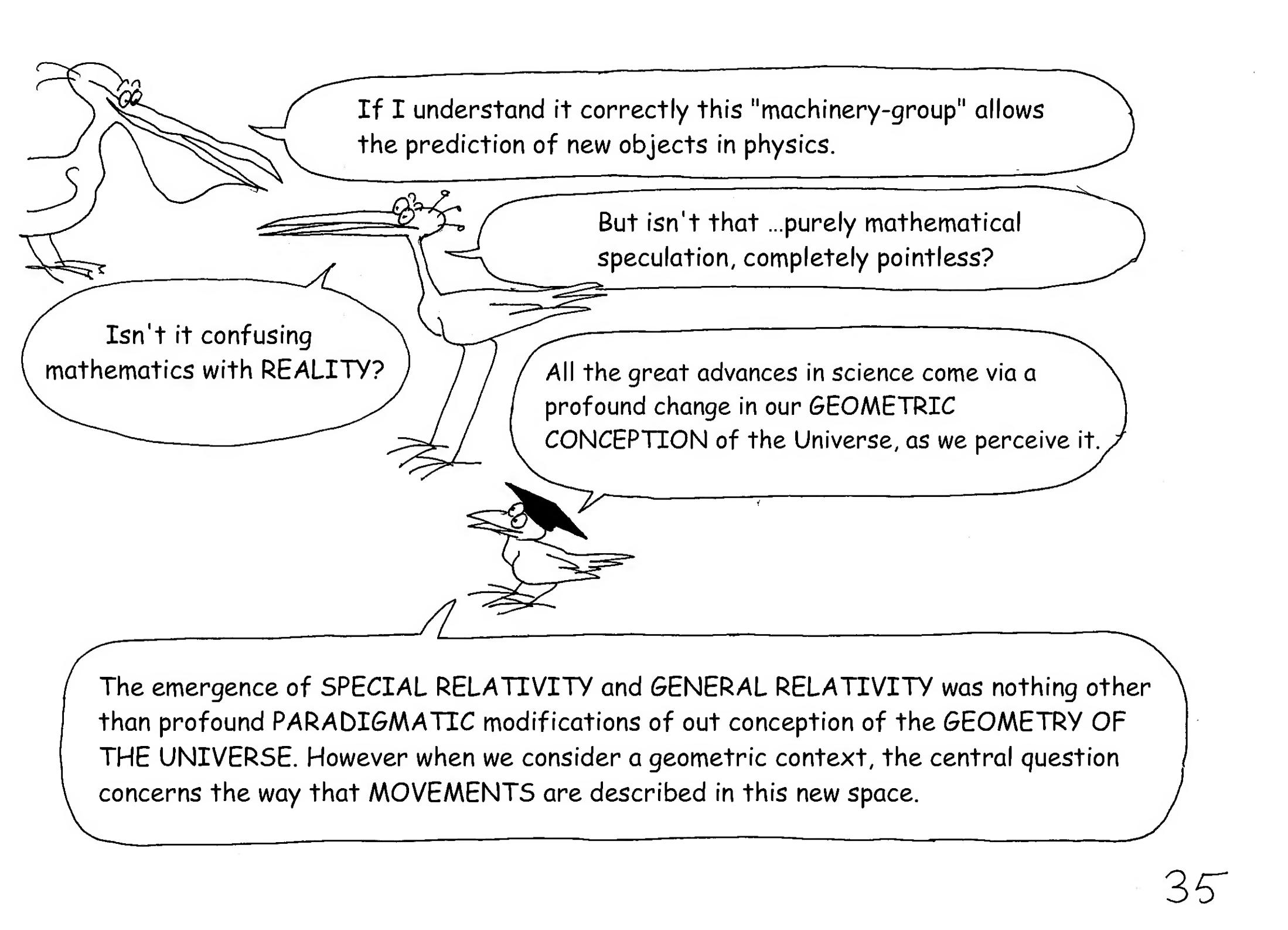
# GOING THROUGH WALLS

Matter, at ordinary densities, is made of tiny atoms separated by a lot of space. It all holds together because of ELECTROMAGNETIC FORCES, the same forces that stop your posterior from going through the chair on which you're seated while reading this book, even though your seat and your posterior are also made of minuscule atoms separated by a lot of space. If we suddenly suppressed the electromagnetic forces, which negotiate through the game of photon exchange (\*) and which have positive energy, you'd immediately pass through your chair, then the floor and fall towards the centre of the Earth, now only influenced by the FORCE OF GRAVITY.

As these two types of matter repulse each other, any structure made up of negative mass would be subject to an ANTIGRAVITATIONAL effect from the Earth. This structure would also be able to cross any type of barrier of matter. It would be invisible to our eyes and undetectable by our measuring and observation instruments. The opposite would also be true - Passengers on a ship made of negative masses could cross through Earth without seeing it.

*The Management.*

(\*) that we qualify as VIRTUALS in QUANTUM FIELD THEORY



If I understand it correctly this "machinery-group" allows the prediction of new objects in physics.

But isn't that ...purely mathematical speculation, completely pointless?

Isn't it confusing mathematics with REALITY?

All the great advances in science come via a profound change in our GEOMETRIC CONCEPTION of the Universe, as we perceive it.

The emergence of SPECIAL RELATIVITY and GENERAL RELATIVITY was nothing other than profound PARADIGMATIC modifications of our conception of the GEOMETRY OF THE UNIVERSE. However when we consider a geometric context, the central question concerns the way that MOVEMENTS are described in this new space.

Special relativity has melted space and time into the same object: a HYPERSURFACE, A SPACE-TIME where movements are now inscribed according to its GEODESICS - General relativity added curvature. GROUP THEORY encompasses the different types of MOVEMENT that can be inscribed on a given hypersurface and MATHEMATICAL PHYSICS identifies the movement of the objects in this universe, according to the principle:

TELL ME HOW YOU MOVE AND  
I'LL TELL YOU WHO YOU ARE.

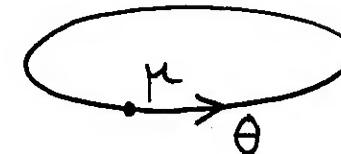
Thus, in a given geometric context, when a possible new type of movement is identified, it suggests the existence of new OBJECTS deriving from this group thanks to the GROUP-TOOL.

But for goodness' sake, give me a concrete example, otherwise your discourse sounds like that of the SUPERSTRING people.

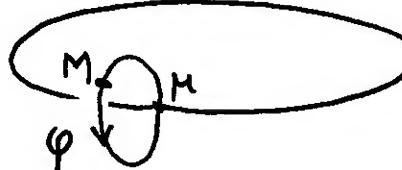
Except that they have neither a geometric context, nor a group, nor movement, nor objects. In short, they don't know WHAT they are saying.

# A FIFTH DIMENSION

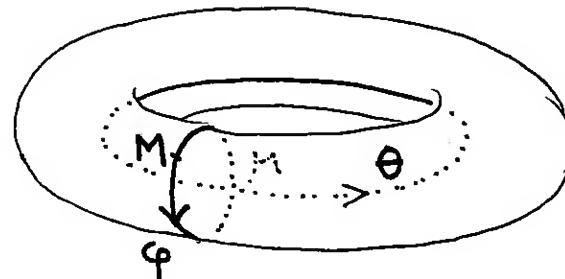
In adding an additional dimension we enrich the geometric context. Take a CLOSED unidimensional universe represented by a simple circle.



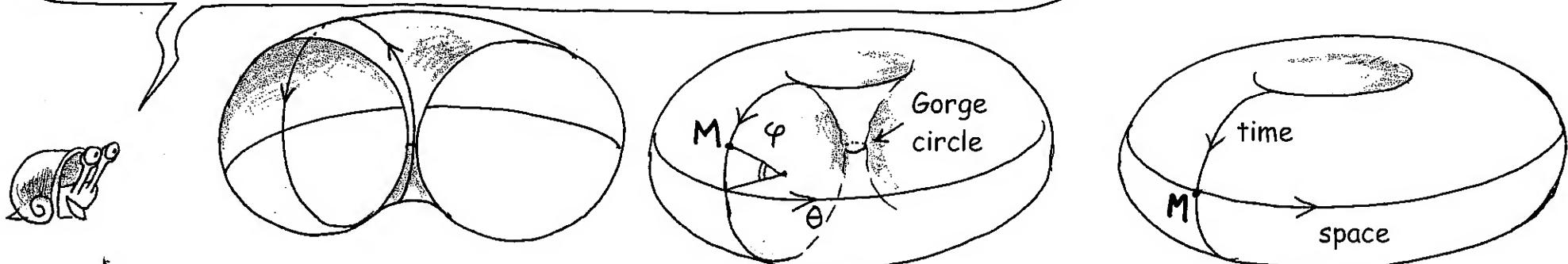
Add another dimension, also closed, on all points of the circle. We'll call it a BUNDLE.



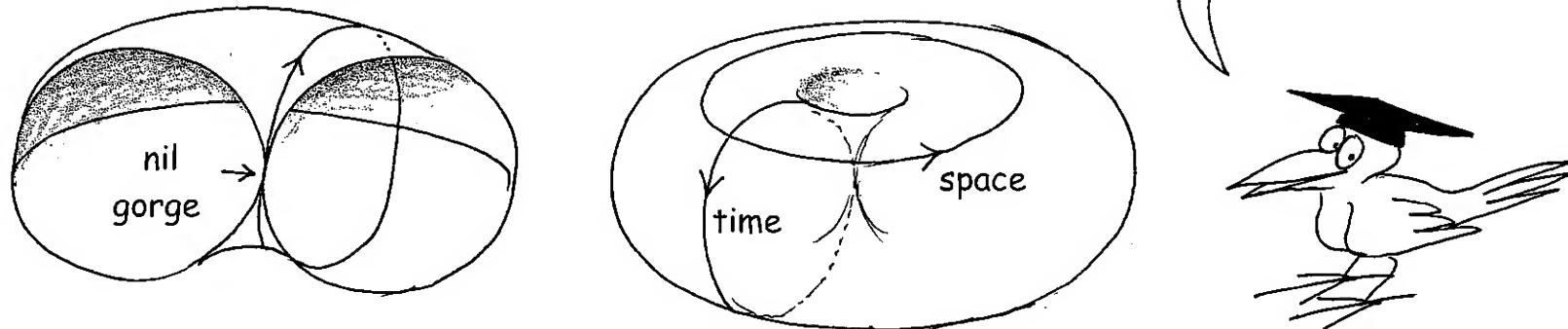
The object obtained, with two dimensions, becomes a TORUS T2



What do we know of the TOPOLOGY (\*) of the space in which we live? We don't even know if it is infinite or closed on itself - We can imagine, for example, a 2D space-time which has the topology of a TORUS  $T^2$ .

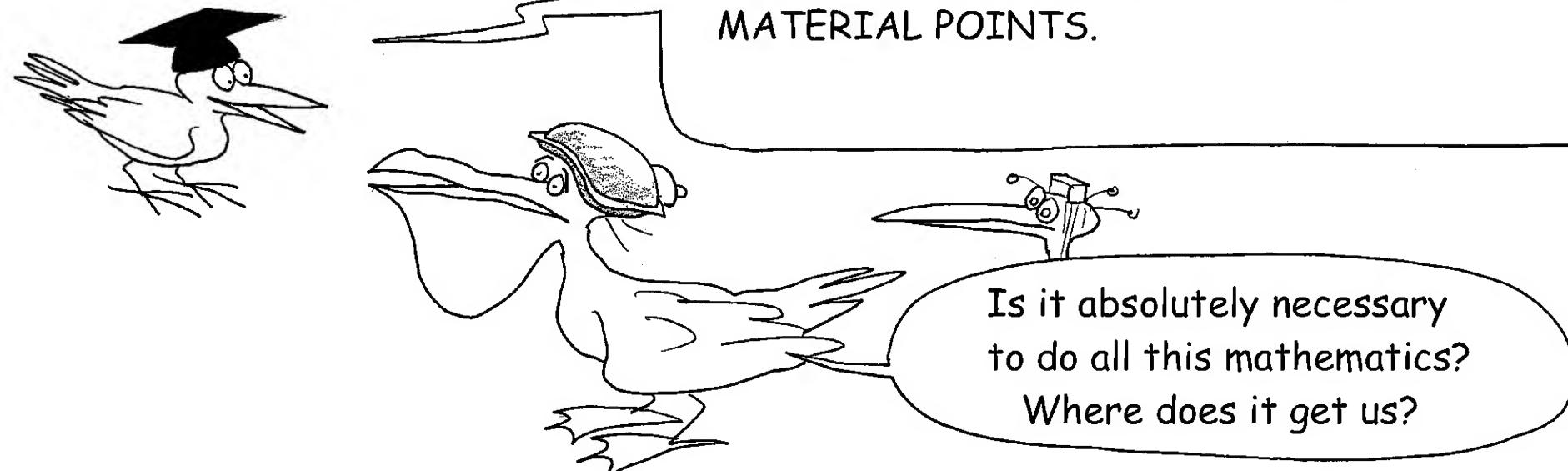


At every point of a circle representing time (x) we place another circle (O) that is supposed to represent a closed space (\*). The gorge circle is supposed to represent the BIG BANG and a BIG CRUNCH together, without "initial singularity". In a case where a singularity is desired absolutely we can consider a TORUS WITH A NIL GORGE.



(\*) If we had wished, we could have placed a "time circle" on every point of a "space circle".

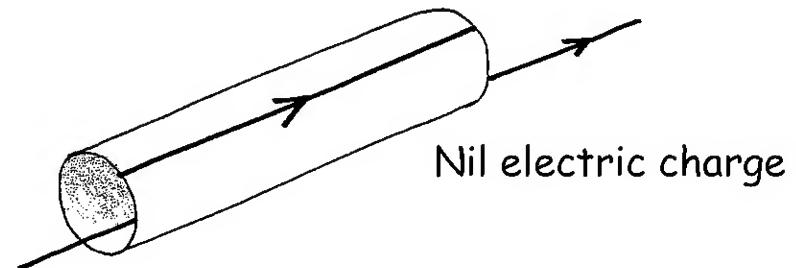
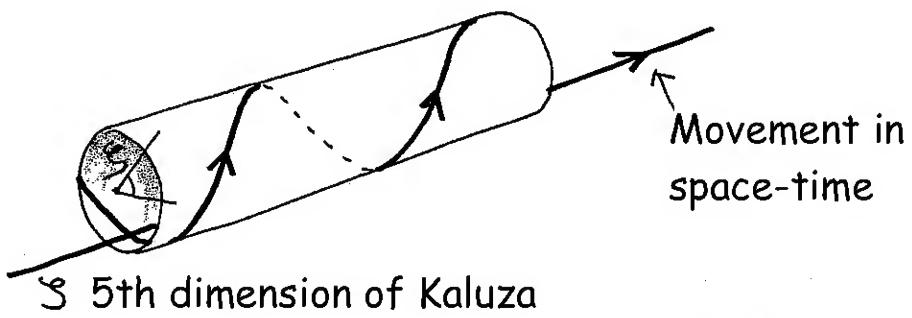
On each point of my 2D space-time I can add a new circle and create a T3 TORUS. We then go from a 2D space to a 3D space by operating a BUNDLING. Such a transformation from a 2D to a 3D space can be represented by a doormat. At each point  $(x, y)$  of a planar object we fix a Z BUNDLE. The 3D object thus obtained is called a BUNDLE. We have to imagine a world in which the fibres of the doormat close in on themselves (which will make the doormat useless). In short, we can imagine that our 4 dimensional space, one of time and three of space, is a T4 TORUS. We can repeat the operation and make a new fibre "grow" at each point, which closes in on itself. We obtain a T5 TORUS. And it is in this new space of 5 dimensions that we can inscribe the MOVEMENTS of our RELATIVIST MATERIAL POINTS.



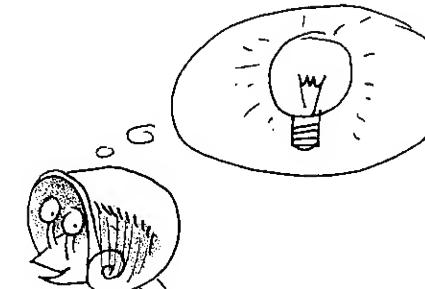
# KALUZA SPACE

We've already said that PHYSICS is a GEOMETRY. Well, inscribing the movement of a particle in a five dimensional hypersurface is, in fact, equivalent to considering that the relativist material point is endowed with an ELECTRIC CHARGE  $e$ . The fact that this fifth dimension, called "Kaluza", is closed on itself means that the electric charge can only take whole values (GEOMETRIC QUANTIFICATION). We can shrink the dimension of a space to a single point. Then the movement of the electrically charged relativist material point will correspond to a spiral curve.

*The Management*

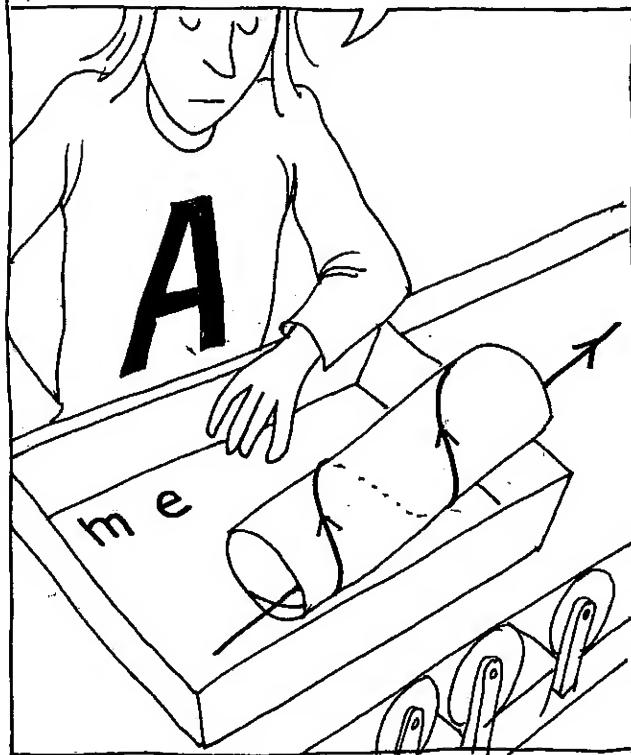


OK, I understand. The direction of the curved spiral corresponds to the sign of the ELECTRIC CHARGE.



Could there be a group behind that?

OK I start a movement, mass  $m$  and the charge  $e$ .



Of course, where there is geometry there is an underlying group (\*).

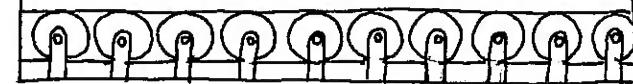
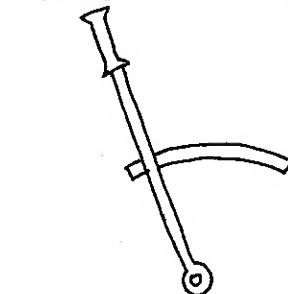
There's a lever which allows the inversion of the spiral's direction. Look at the effect on the mass and the charge.

orthochronic

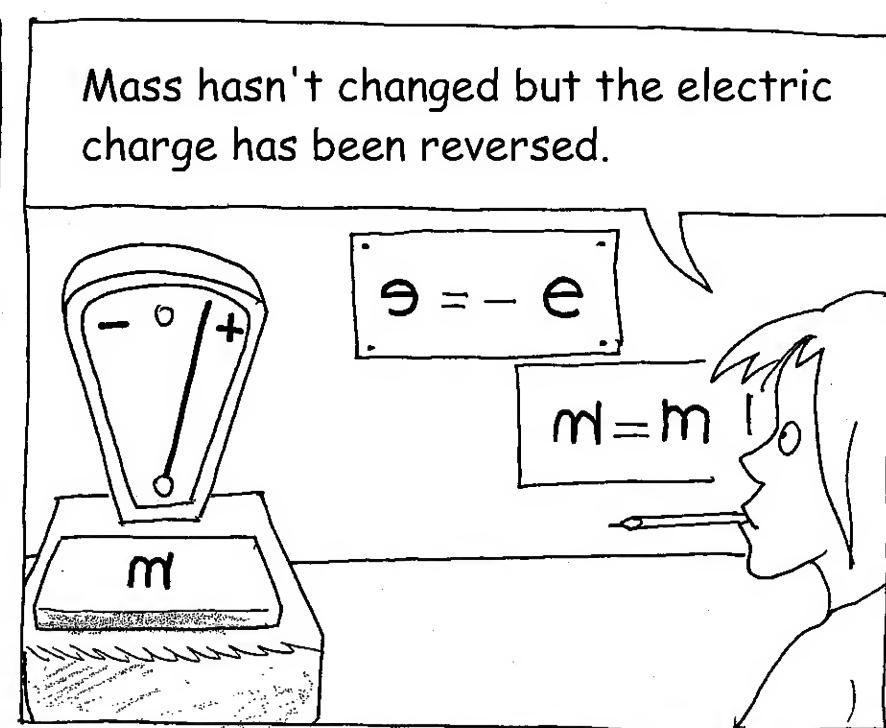
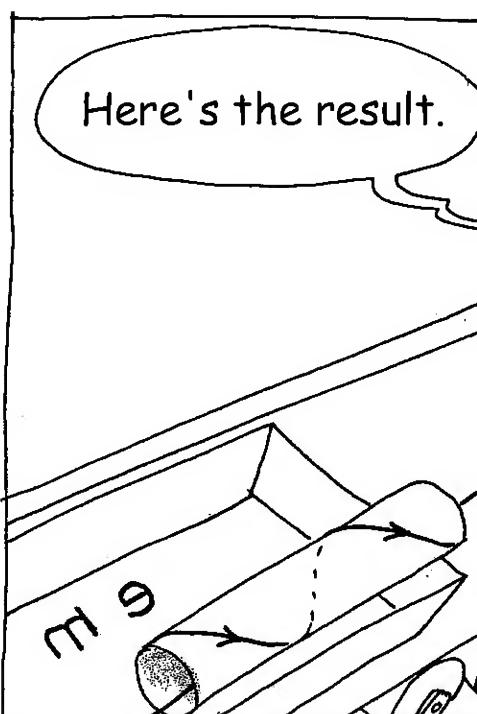
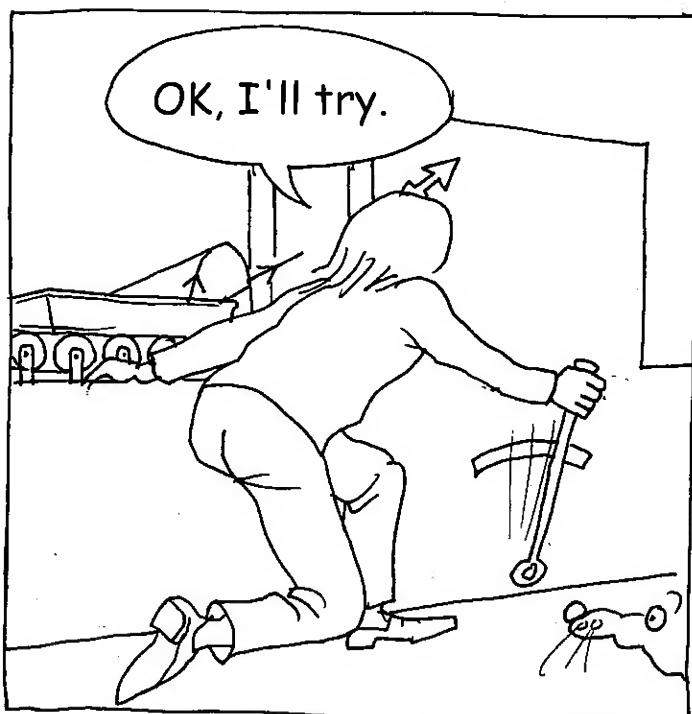
retrochronic

DIRECT

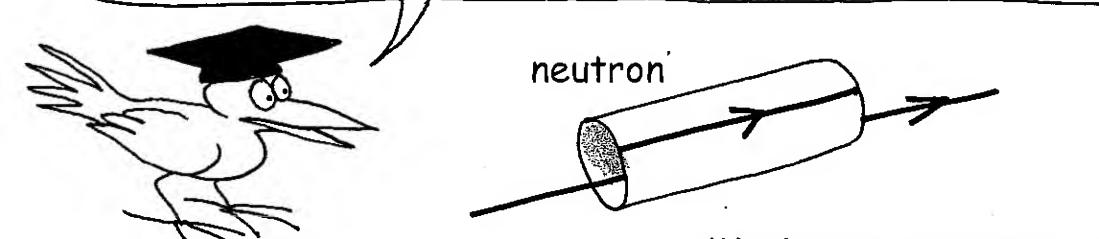
REVERSE



(\*) The "extended Poincaré group". See the Annex 4



This inversion of electrical charge evokes immediately the transformation matter  $\rightarrow$  antimatter. But according to this schematic model the neutron, whose electrical charge is nil, will be its own antiparticle, which isn't true. In fact particles have, on their "identity card", a certain number of "quantum charges" (hadronic, leptonic etc), the electric charge  $e$  was just one of these charges among all the others. The transformation of a particle of matter into its antiparticle consists of inverting all its quantum charges (\*), including its electric charge if it isn't zero. It should be remembered that if the charges change, the mass isn't modified.



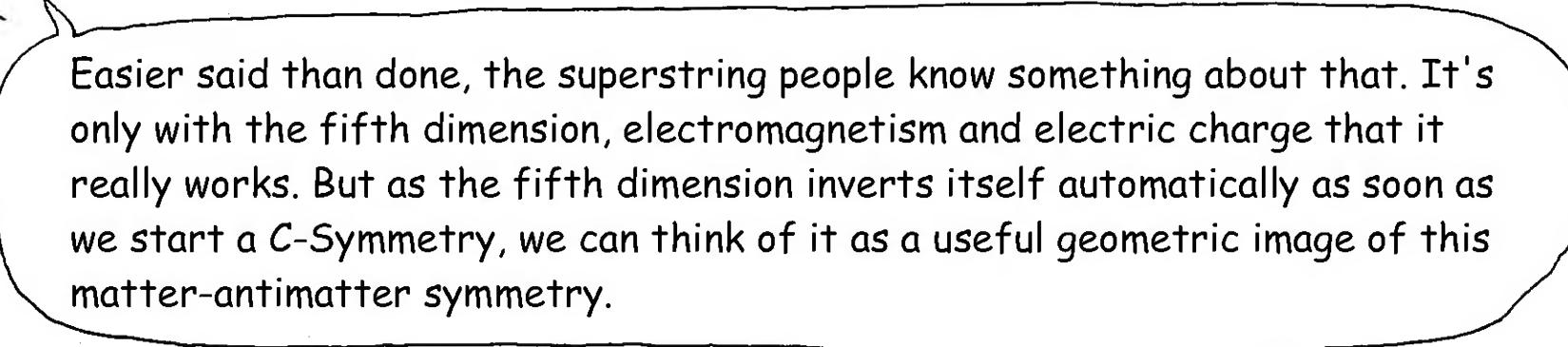
(\*) Charge union or C - SYMMETRY



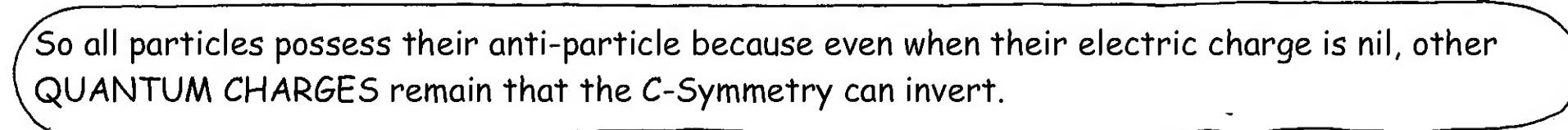
In short, antimatter has positive mass.



Why not add some more dimensions in order to make every aspect of the particles appear?



Easier said than done, the superstring people know something about that. It's only with the fifth dimension, electromagnetism and electric charge that it really works. But as the fifth dimension inverts itself automatically as soon as we start a C-Symmetry, we can think of it as a useful geometric image of this matter-antimatter symmetry.



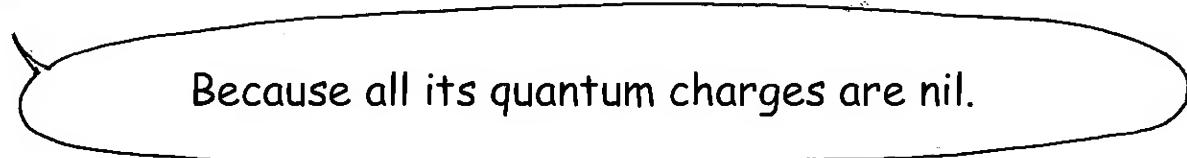
So all particles possess their anti-particle because even when their electric charge is nil, other QUANTUM CHARGES remain that the C-Symmetry can invert.



The exception is the PHOTON.



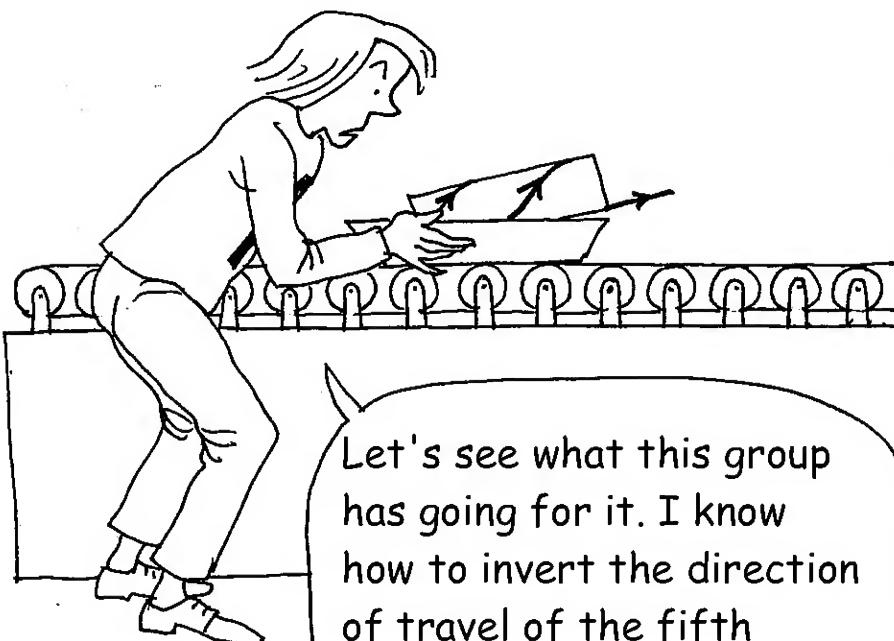
Why?



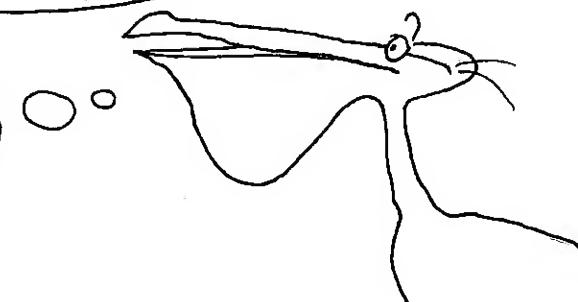
Because all its quantum charges are nil.

So what remains? Nothing?

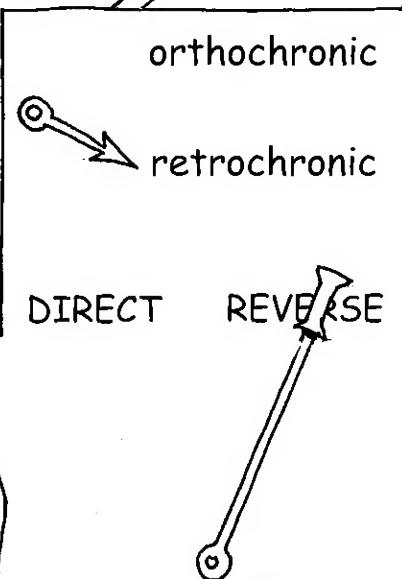
No, its energy  $E = \hbar\nu = \frac{\hbar}{\tau} (*)$  remains



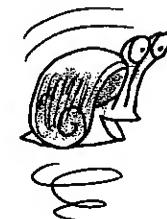
Doesn't this lad ever stop?



Let's see what this group has going for it. I know how to invert the direction of travel of the fifth dimension and of travel through time.



And what does that give?



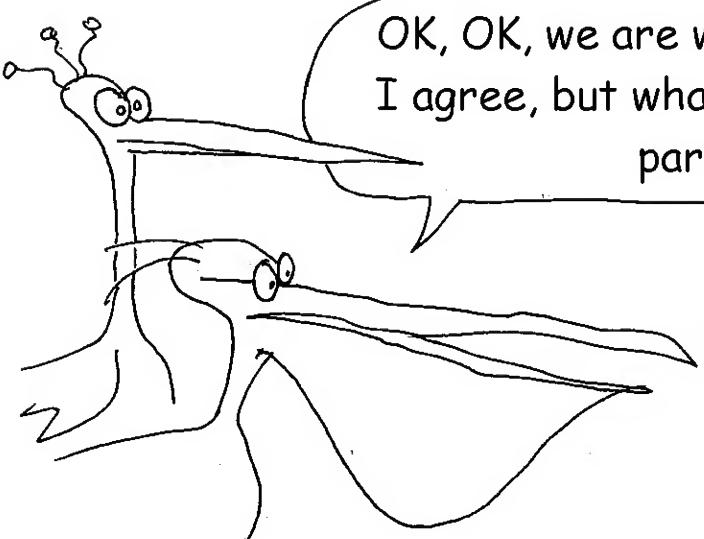
(\*) and its spin, see the Annex.

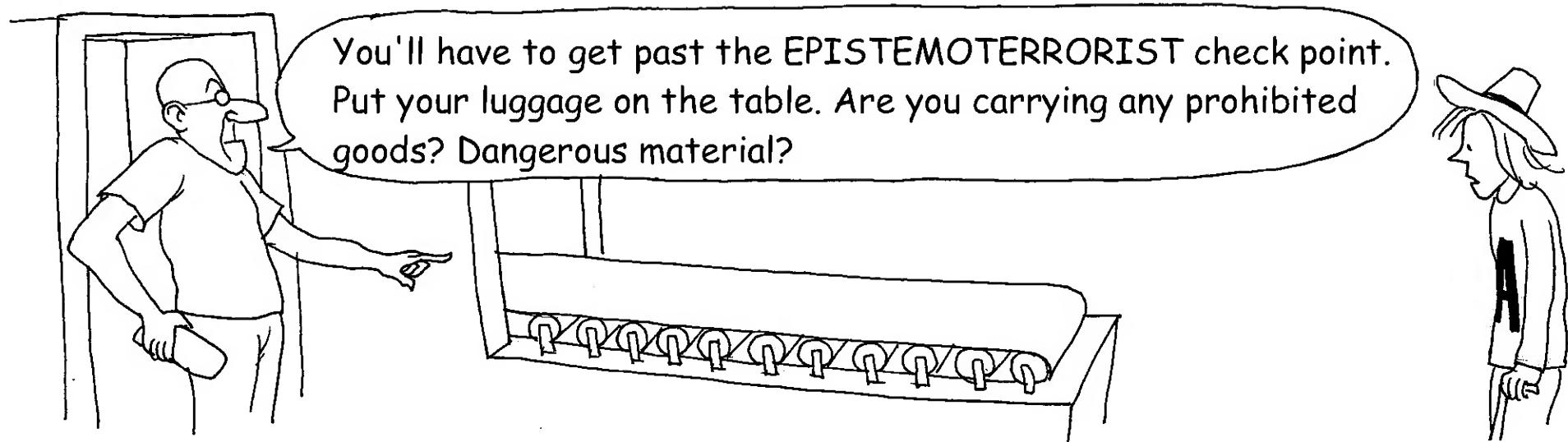
Electric charge  $\bar{e}$  inverted and mass  $\bar{m}$  inverted. That means that I obtain anti-matter with negative mass and energy. In other words, the matter-anti-matter symmetry also exists in this world of negative masses. But ignoring the fact that masses and energies are inverted, this other matter, what could it look like?

Conclusion: The MATTER-ANTIMATTER DUALITY can also be found in this WORLD OF NEGATIVE ENERGIES where a particle of negative mass can "annihilate itself" with its antiparticle, also of negative mass, giving  $\bar{\phi}$  photons of negative energy.

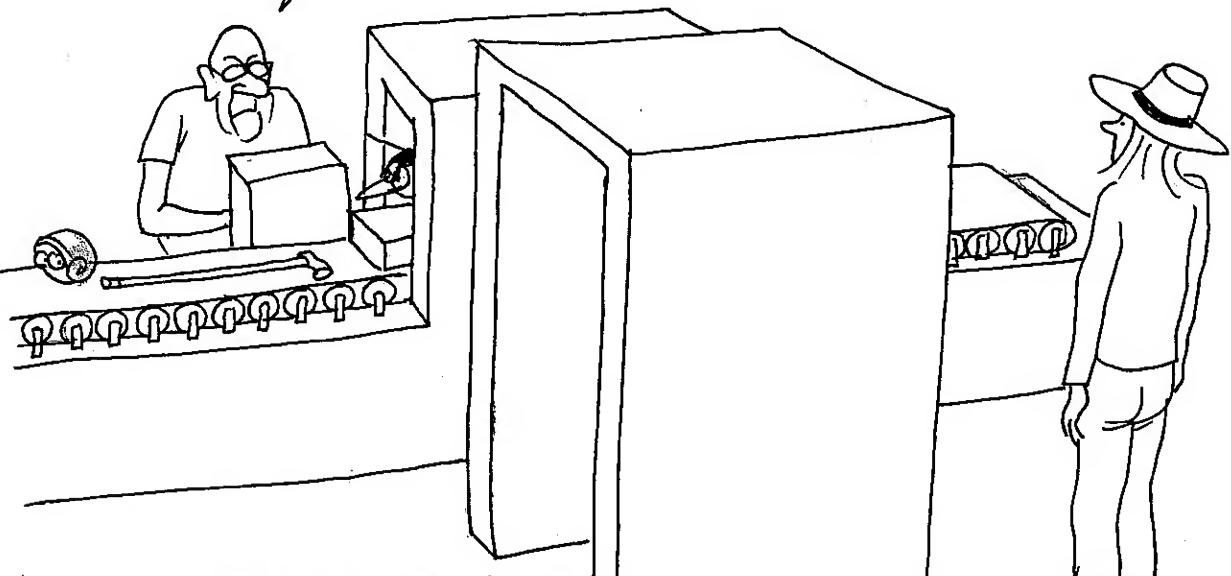
OK, OK, we are wading through real fiction here  
I agree, but what do these negative energy  
particles look like?

We find  $\bar{p}$  protons,  
 $\bar{e}$  electrons,  $\bar{n}$  neutrons,  
 $\bar{\nu}$  neutrinos etc, all endowed  
with negative energy.

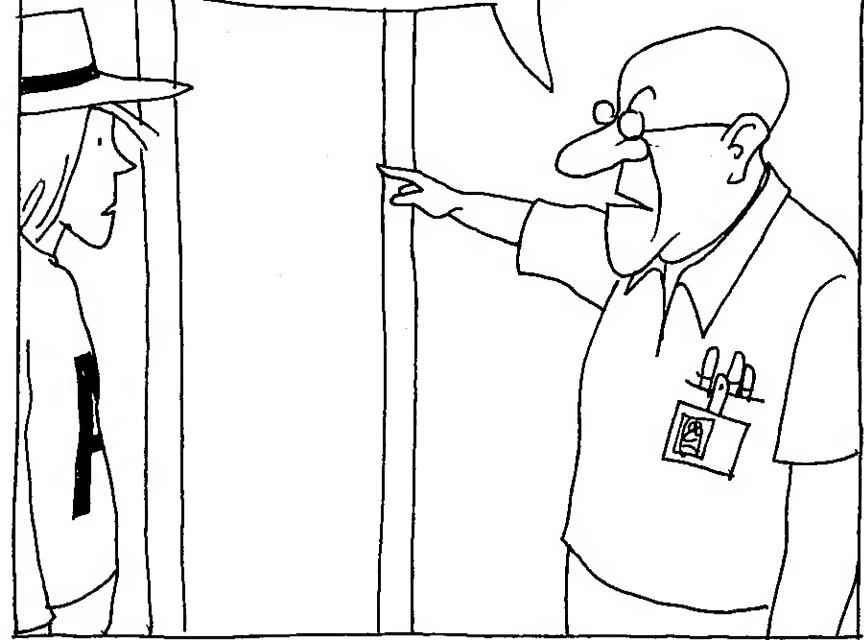




No negative energy particles?

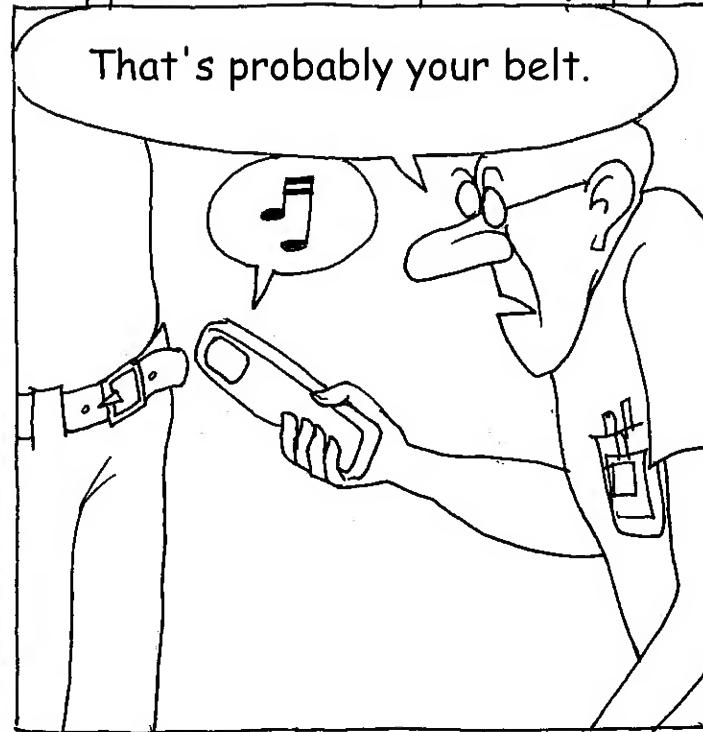
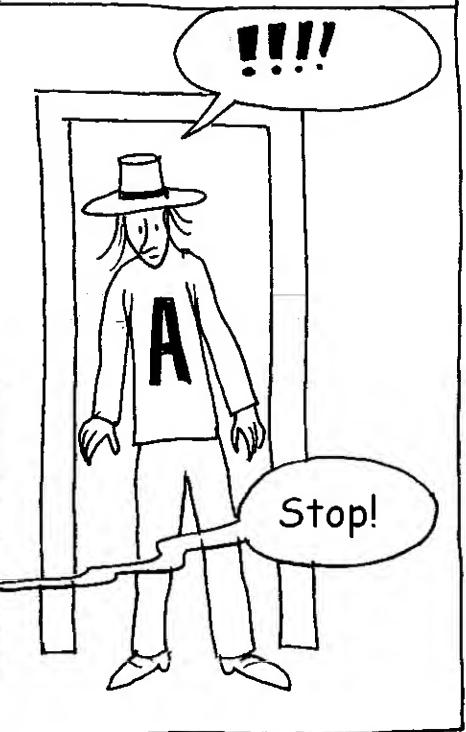


OK. Now go through the scanner.

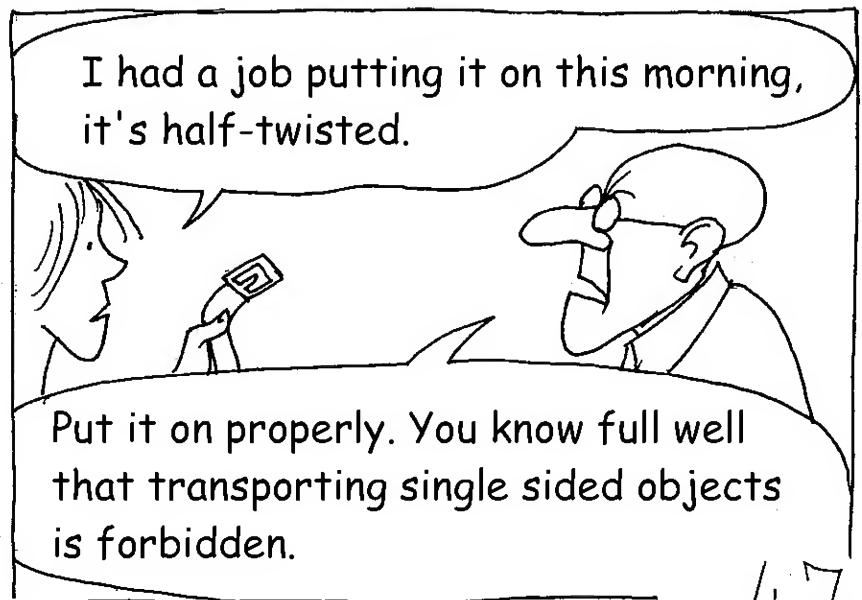


!!!!

That's probably your belt.



I had a job putting it on this morning, it's half-twisted.



Put it on properly. You know full well that transporting single sided objects is forbidden.

How was your journey?

We have a first answer = moving backwards in time is simply having negative mass and energy.

I'm happy to hear it - but what about going in the past future direction, what does that mean?!?

It means your energy is positive that's all.

Hmmmm..

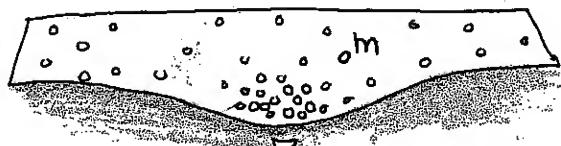
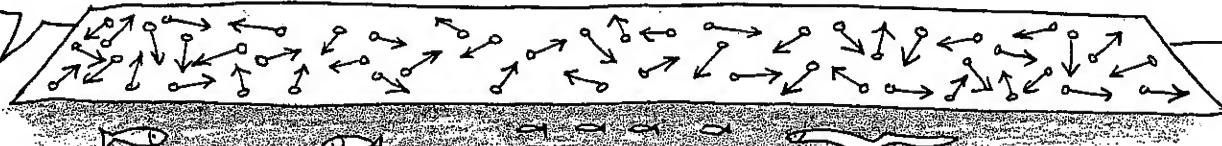
We still have to build a cosmological model where the universe is filled with positive masses and negative masses - If, for a reason that it would be helpful to justify, negative mass was greater than positive mass, that would provoke an **ACCELERATION**. That will be it then, the mysterious **DARK ENERGY**.

Let's leave that discussion until later and examine the behaviour of this cosmos with two populations.

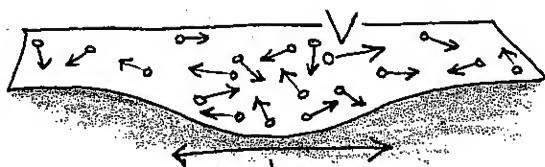
(\*) The existence of negative masses and energies: "Structure des systèmes dynamiques" 1972, downloadable at [www.jmsouriau.com](http://www.jmsouriau.com), more precisely page 198, equation 17.67

# THE LARGE SCALE STRUCTURE : EXPLAINED

In the album A THOUSAND MILLION SUNS (1986) we presented a fundamental problem in astrophysics: GRAVITATIONAL INSTABILITY or JEANS' INSTABILITY (pages 12 to 23). We'll return to this idea while modifying the model a little. Matter will be represented by lead shot spread over a big, flexible rubber mat covering an area of water. The lead shot can move freely on the surface with a random speed representing the THERMAL AGITATION RATE (\*) of this 2D milieu.



$$t_a = \frac{1}{\sqrt{4\pi G \rho}}$$



$$t_d = \frac{L}{V}$$

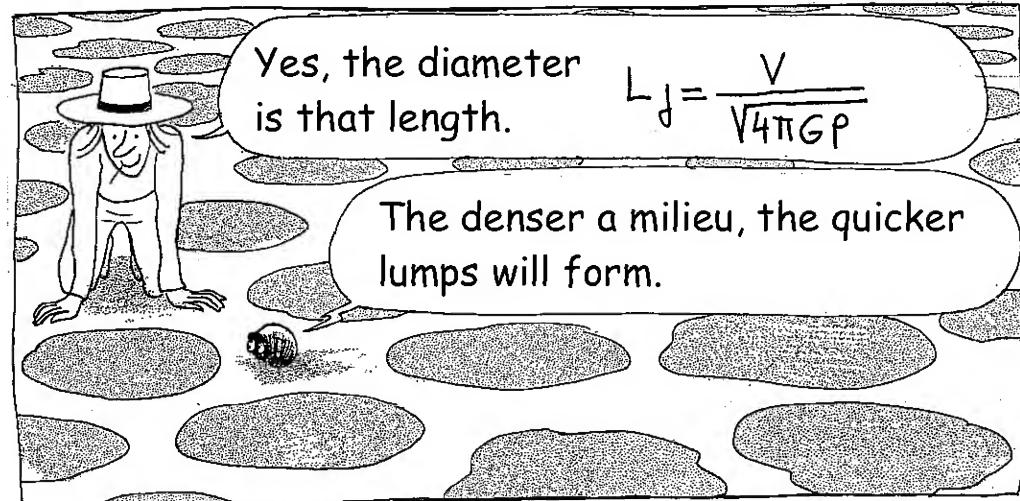
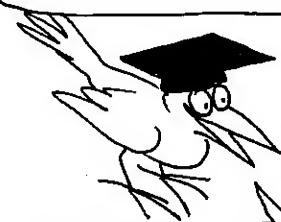
When chance causes an assembly, a local superdensity of matter, this attracts the matter around it (the ACCRETION phenomenon). The characteristic growth time  $t_a$  of this perturbation is in  $\frac{1}{\sqrt{\rho}}$  where  $\rho$  is density.

Inversely this "lump" will tend to disperse in a time

$$t_d = \frac{L}{V}$$

(\*) ABSOLUTE TEMPERATURE is defined as  $\frac{3}{2} kT = \frac{1}{2} mV^2$  where  $k$  is Boltzmann's Constant  $(1.38 \cdot 10^{-23} \text{ MKSA})$

The lumps that will appear will be those whose diameter is equal to the Jeans' (\*) distance which, statistically, have more chance of appearing than bigger ones.



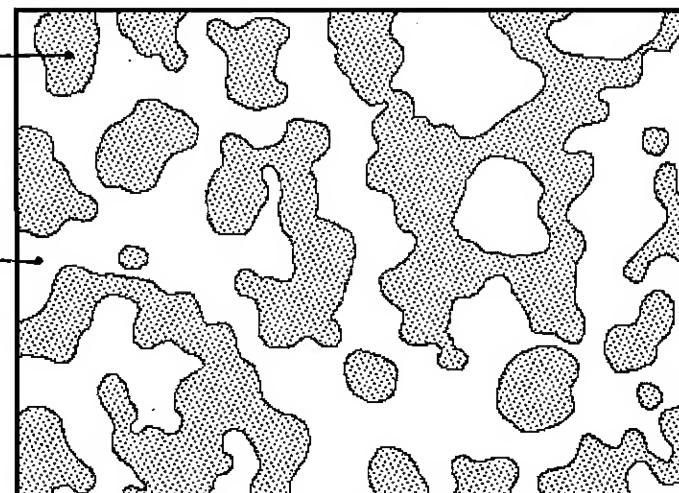
As negative masses attract each other they will also form their own "lumps". If we start with a milieu where negative and positive masses have the same densities and the same thermal agitation rate, they will simply share the available space because they repulse each other.

Like people who can't bear to be near each other.

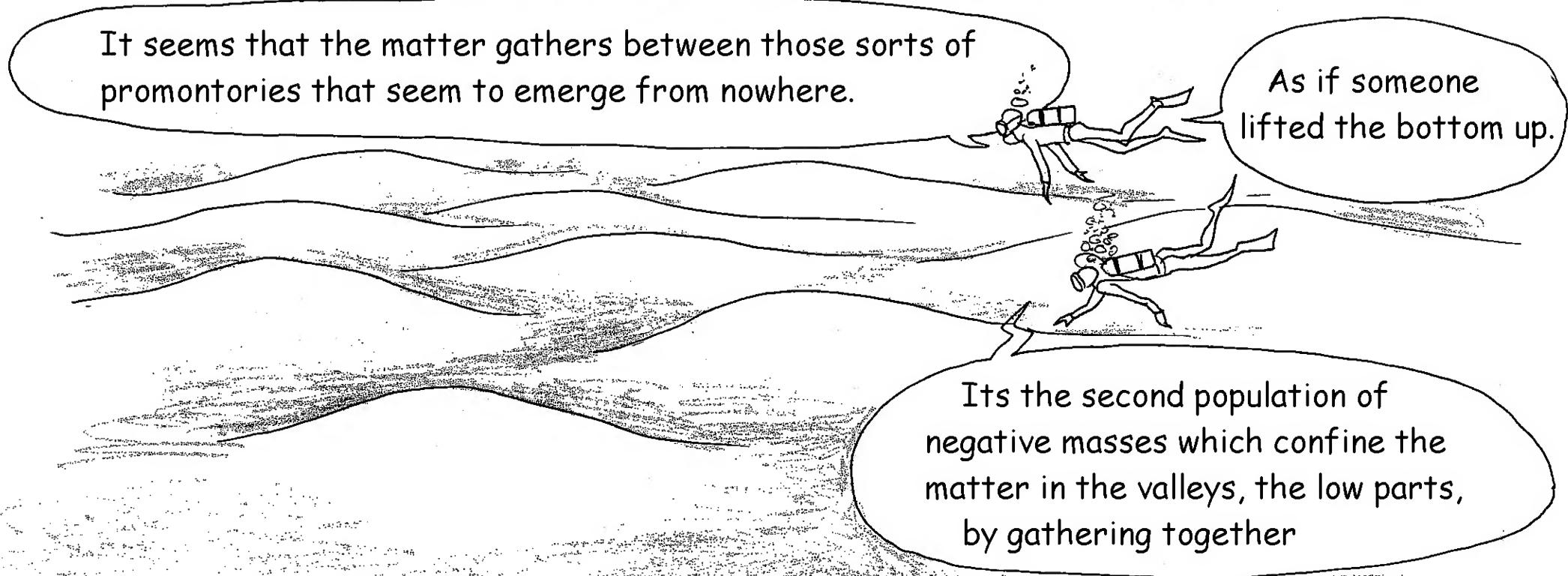
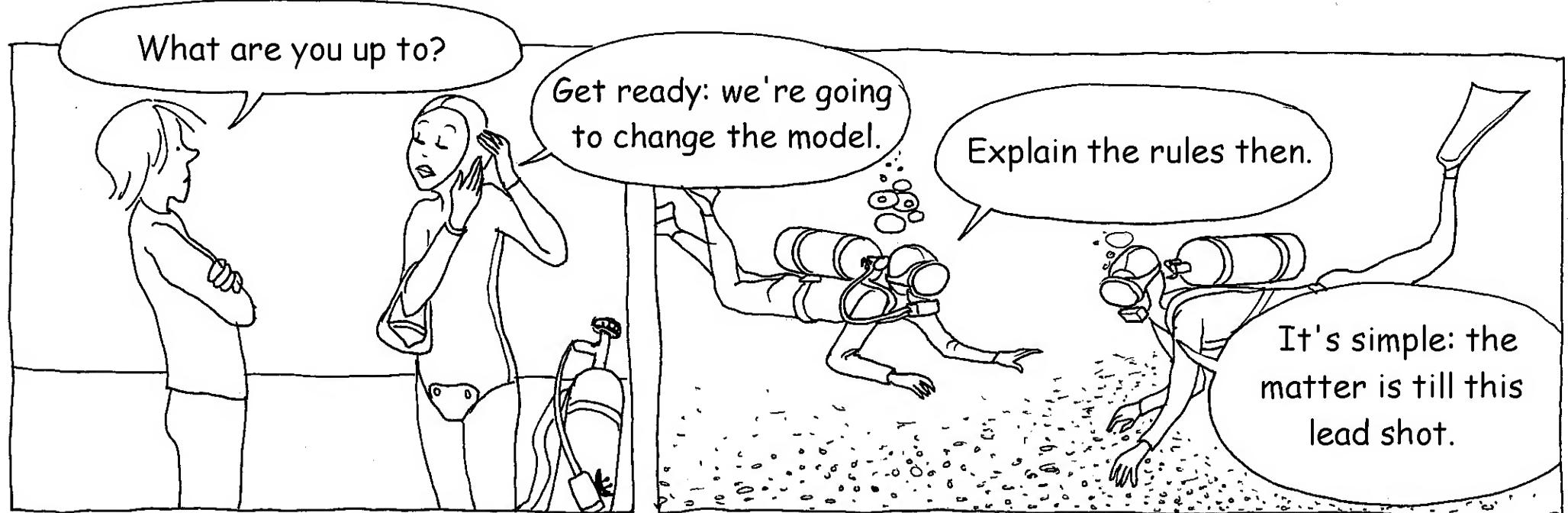


matter of positive mass

matter of negative mass

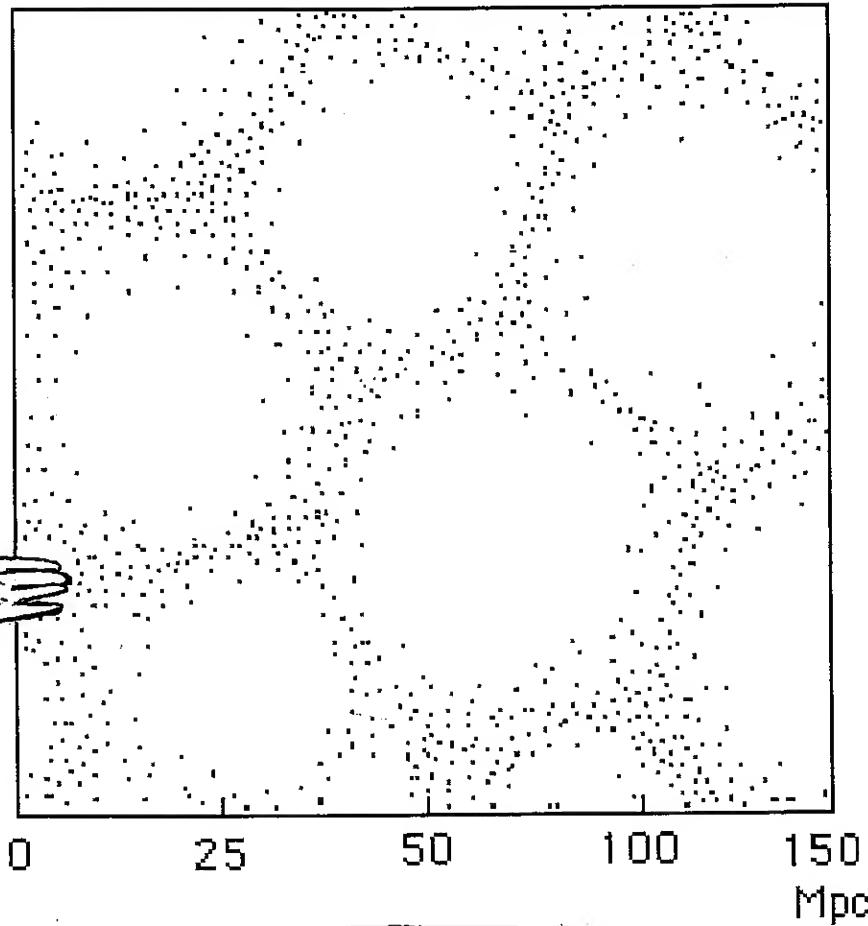


(\*) Sir James Jeans, English astronomer (1877-1946)

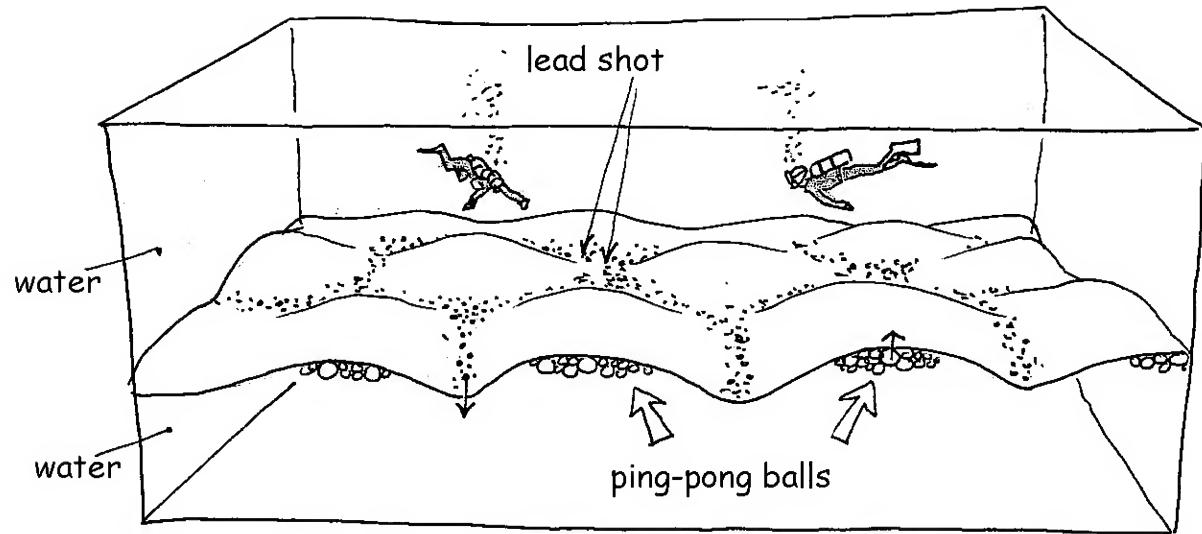


A nice rundown

J.P.Petit : Twin Universe Cosmology  
Astronomy and Space Science 226 : 273-307, 1995



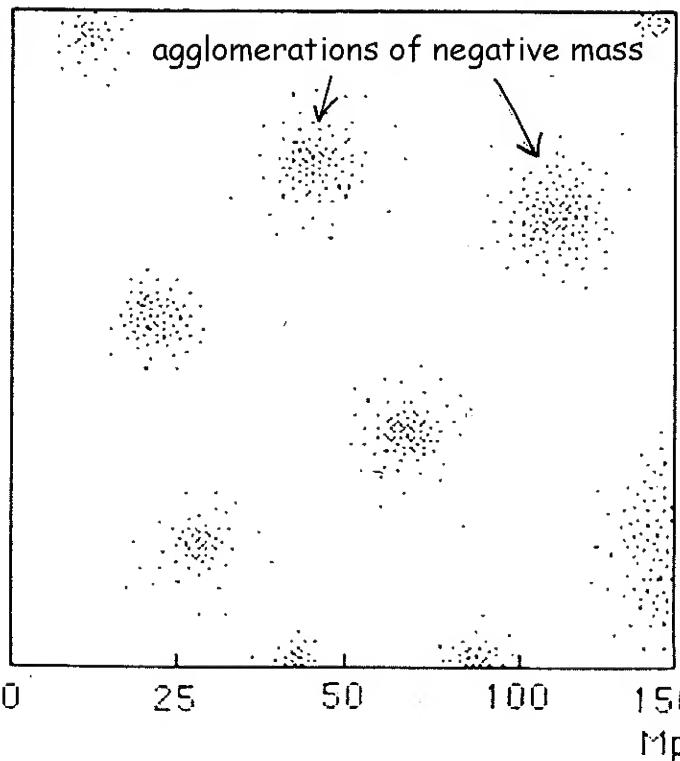
Each of these "links" is about a hundred million light years in diameter.



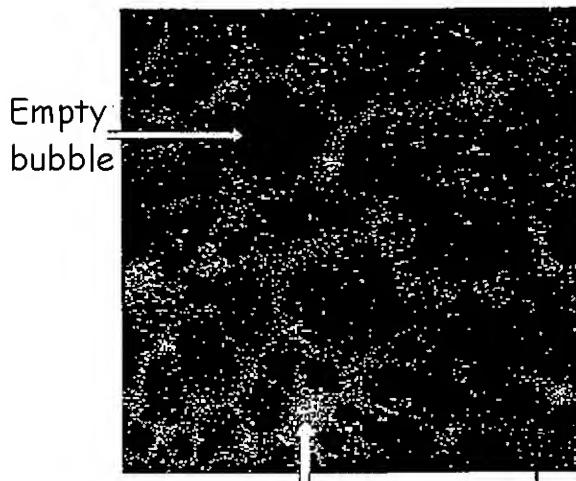
This model is there to illustrate the idea of CONJOINED GRAVITATIONAL INSTABILITY which will affect a mixture of positive and negative masses if the density  $\rho$  of the negative mass is the greater.

It will form agglomerations more rapidly by imposing its structure on the large scale

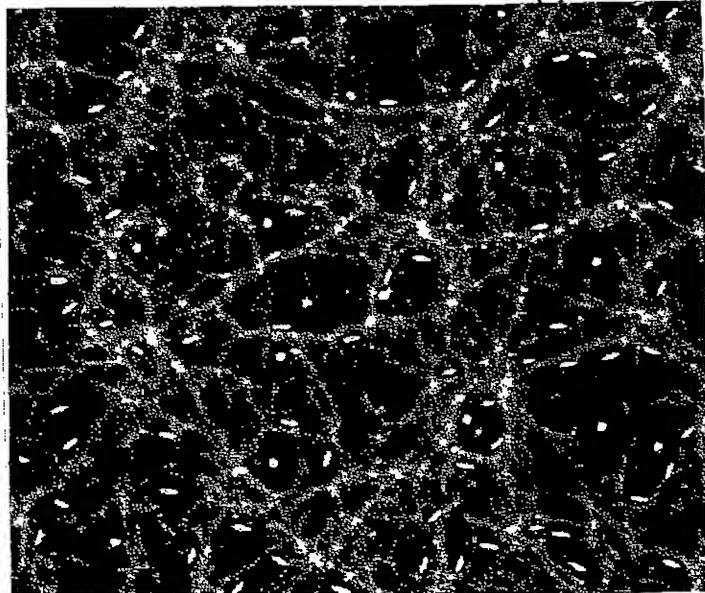
universe. The rubber membrane evokes their invisibility for an observer made of positive mass. - In case, on the left, here is what an observer made of negative mass would see. He wouldn't see our own matter, which is distributed in a LACUNAR manner, a PROVEN OBSERVATIONAL fact, like "jointing soap bubbles" around "voids" of hundreds of million light years diameter. - Numerical simulations undertaken in 1992 with a mixture of the two matters led to images in conformity with observations while the classical model, even if resorting to COLD DARK MATTER gives a FILAMENTAL STRUCTURE WHICH DOESN'T AGREE WITH OBSERVATION (following page).



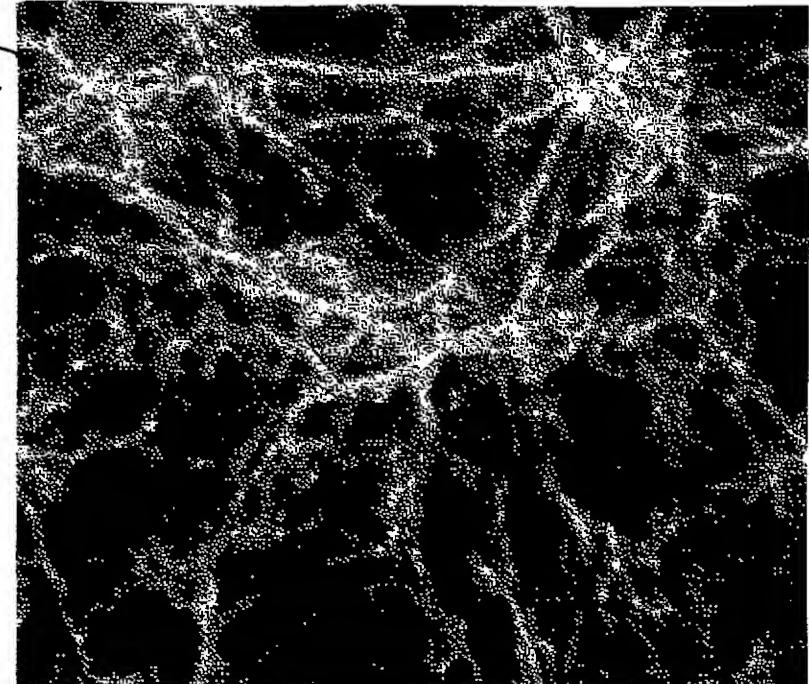
# SCIENTIFIC SURREALISM



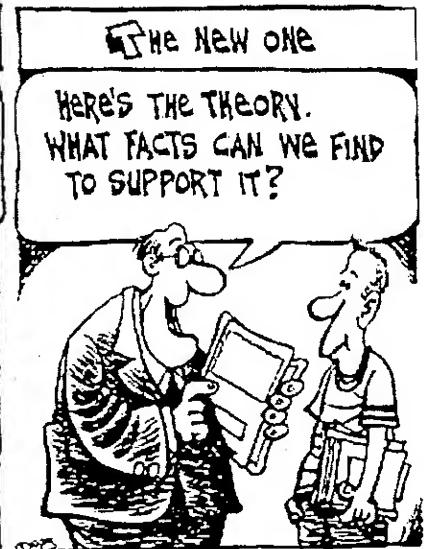
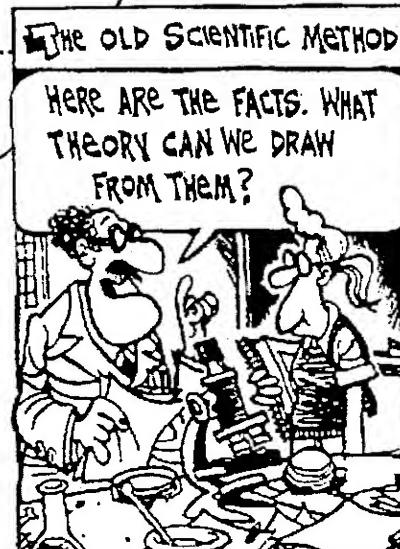
On the top left the VISIBLE PART of the Universe, whose resolutely LACUNAR aspect is confirmed from year to year. On the bottom left, the INVISIBLE PART deduced from the decoding of micro-effects of the gravitational lens. On the top right the result of simulations using COLD DARK MATTER, which agree with the second "observations". All that remains is to map DARK ENERGY...

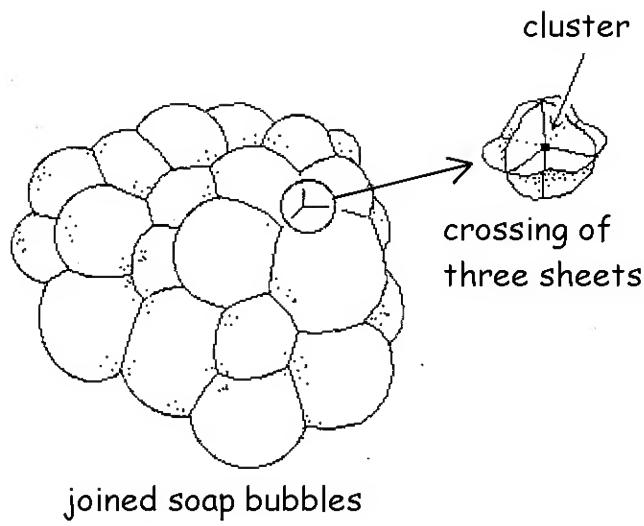


Map of dark matter



Simulation: The Universe at 2 million years old





joined soap bubbles

You are hanging on desperately to the miserable 4% of the universe that we see. Be modern for goodness' sake! Look at the fantastic advances that this NEW ASTRONOMY has brought. In any case, you won't get away from an inescapable FACT: the strong effects of the gravitational lens which PROVE THE EXISTENCE OF DARK MATTER.

Joined soap bubbles!

Ah, I think that Higgins brings a new element!

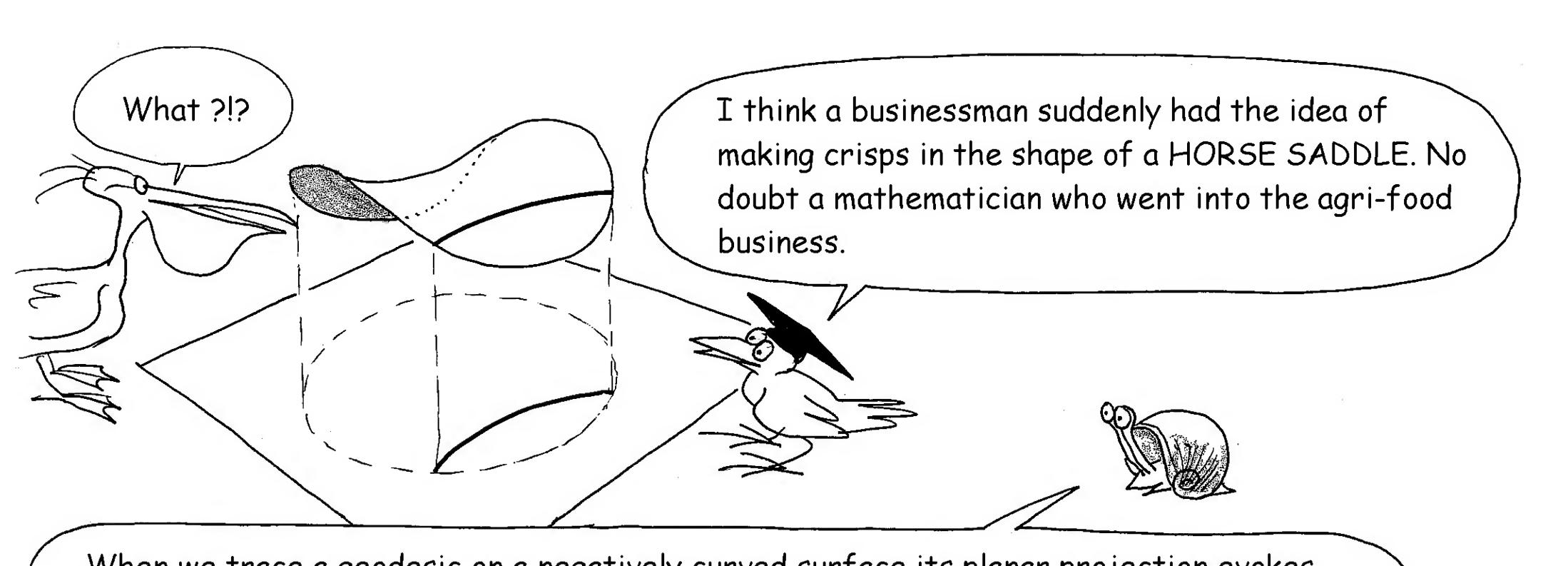


# THE NEGATIVE GRAVITATIONAL LENS EFFECT (\*)



(\*) For the specialist : the negative gravitational lensing effect is an exact solution to Einstein's equation, which nobody had thought about until now. This will be brought up schematically in the annex, for the details see

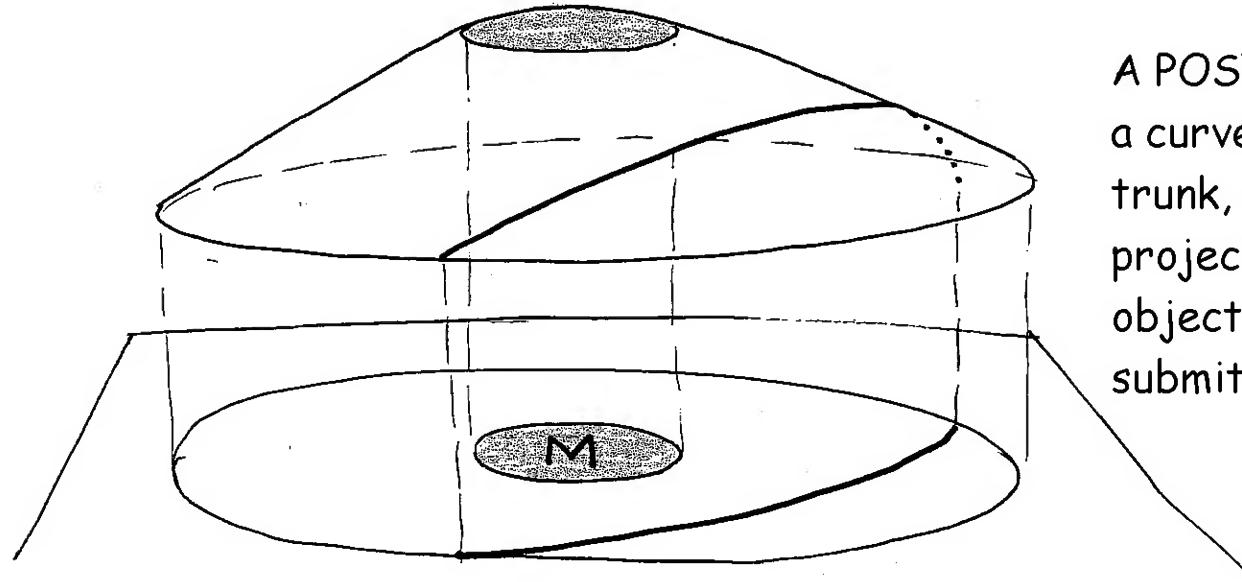
Jean-Pierre Petit : Twin Universe Cosmology : Astronomy and Space Science 226 : 273-307, 1995 et <http://arxiv.org/abs/0801.1477>



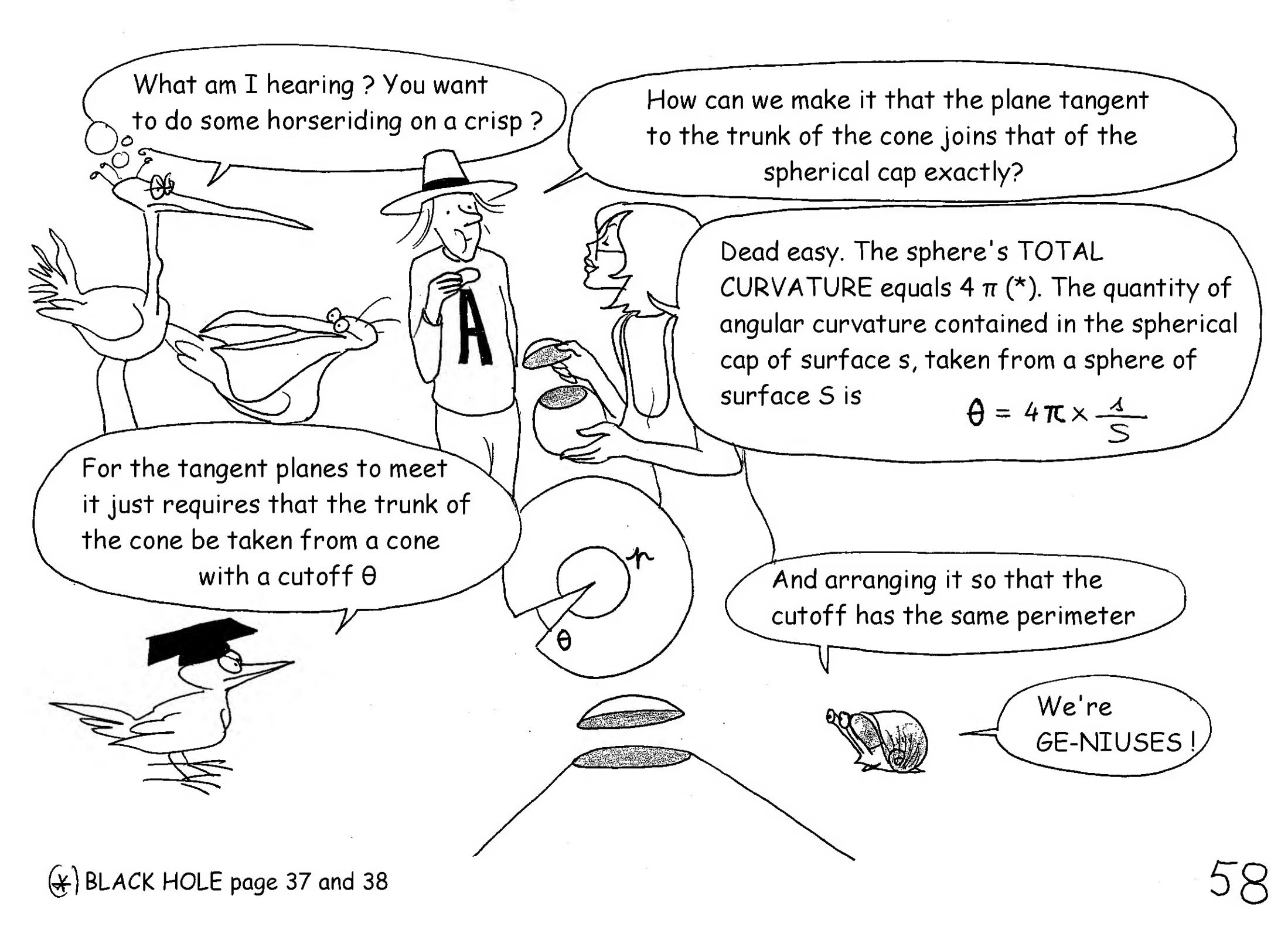
What ?!

I think a businessman suddenly had the idea of making crisps in the shape of a HORSE SADDLE. No doubt a mathematician who went into the agri-food business.

When we trace a geodesic on a negatively curved surface its planar projection evokes a REPULSIVE FORCE. Remember the thing about the BLUNTED CONE.



A POSICONE TRUNK is a spherical cap, a curved surface, completed by a cone trunk, a Euclidian surface. The planar projection gives the impression that an object, on its trajectory, would be submitted to the attraction of a mass  $M$ .



What am I hearing? You want to do some horseriding on a crisp?

How can we make it that the plane tangent to the trunk of the cone joins that of the spherical cap exactly?

For the tangent planes to meet it just requires that the trunk of the cone be taken from a cone with a cutoff  $\theta$

Dead easy. The sphere's TOTAL CURVATURE equals  $4\pi$  (\*). The quantity of angular curvature contained in the spherical cap of surface  $s$ , taken from a sphere of surface  $S$  is

$$\theta = 4\pi \times \frac{1}{S}$$

And arranging it so that the cutoff has the same perimeter

We're GE-NIUSES!

Can we imagine a  
BLUNTED NEGACONE ?

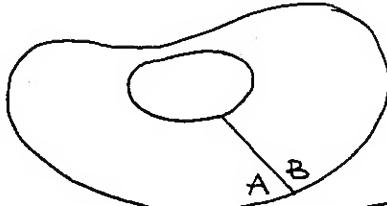
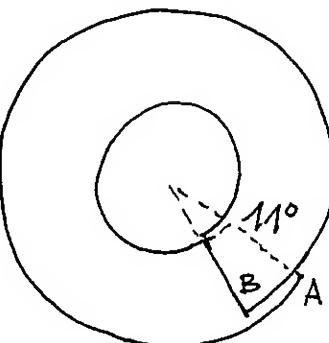
Of course. You just need  
to stick a negachip from  
edge to edge to a negacone.

oh la.

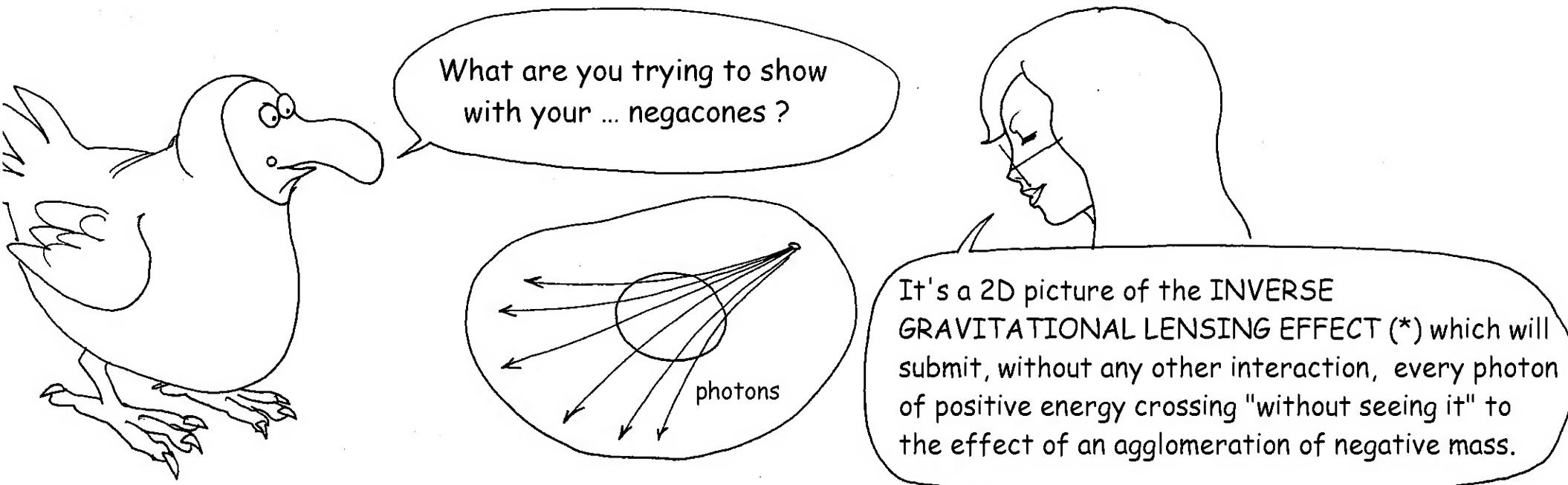
And how do we ensure the  
continuity of the tangent plane ?

A negacone is a disc  
into which we insert  
an angle  $\theta$

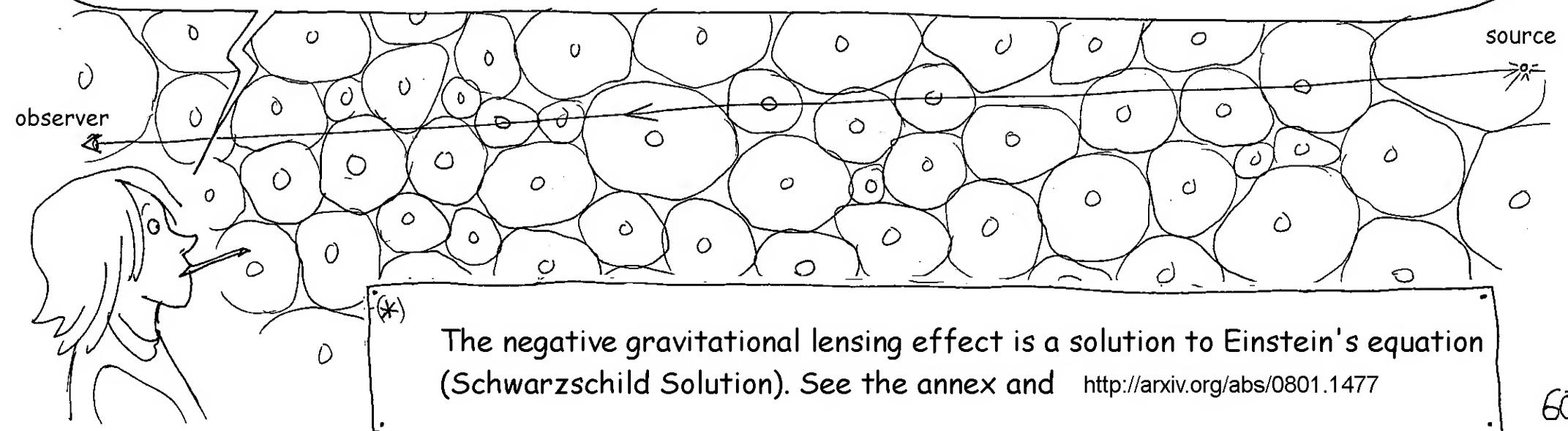
Ah, there there isn't a rule as simple as that for the blunted posicone. We've  
measured the curvatures of the negacrisps and found  $11^\circ$ . It's going to be  
delicate because we haven't found any glue for negacrisps.

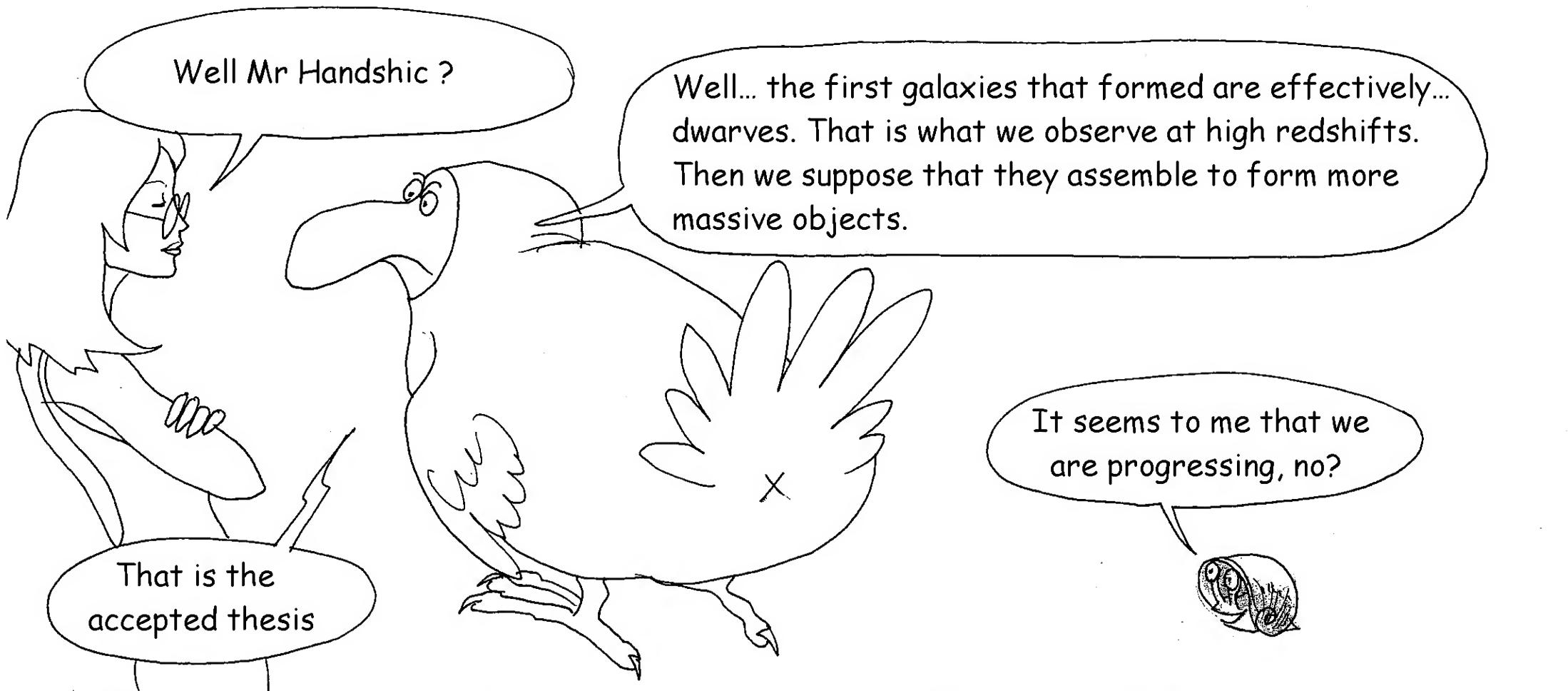


I think that negacrisp makers should print this curvature  
on the packet so that we know what we're eating.



Which means that if, in any direction, we observe objects situated at very great distances, at the extremities of the observable Universe, there are good chances that the light rays cross several agglomerations of negative mass on their way and that this attenuates their luminosity. Thus, logically, images of far distant galaxies with a pronounced redshift should make them appear as dwarves.



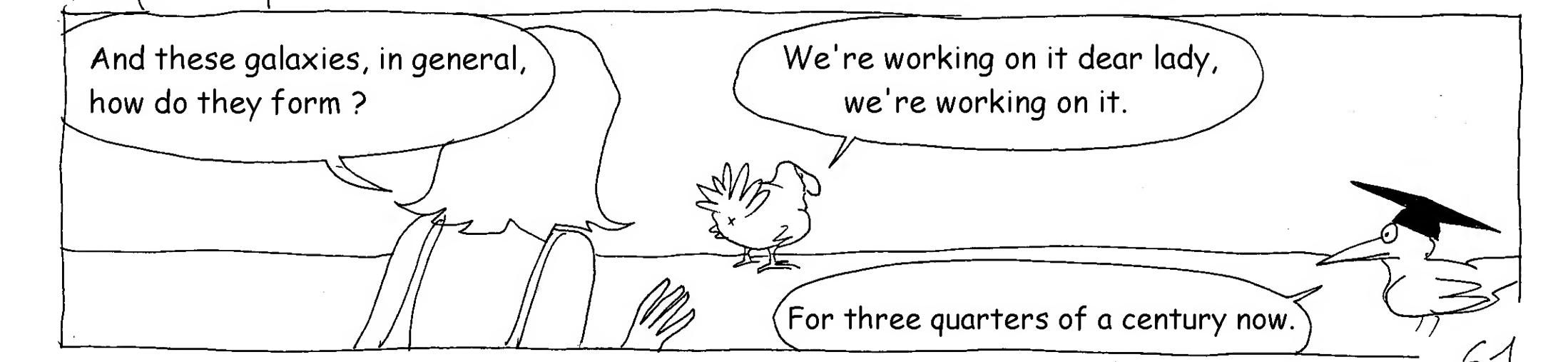


Well Mr Handhic ?

Well... the first galaxies that formed are effectively... dwarves. That is what we observe at high redshifts. Then we suppose that they assemble to form more massive objects.

That is the accepted thesis

It seems to me that we are progressing, no?



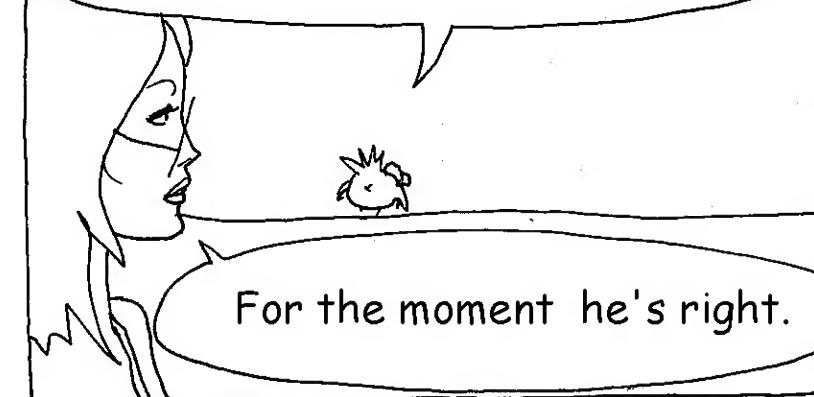
And these galaxies, in general, how do they form ?

We're working on it dear lady, we're working on it.

For three quarters of a century now.



You've scored a point but don't forget, band of imps, that your tale of negative masses does not explain in any way the strong effects of gravitational lensing near galaxies, and especially galaxy clusters.



For the moment he's right.

# HOW STARS ARE FORMED



Before asking how galaxies are formed, we might reflect on the way stars are formed.



Stars: We know more or less how they function. In relation to our ephemeral human lives, and even our civilisations, their development extends over an immeasurably longer time. The key progress, at the beginning of the 20th century, when it was understood that there wasn't an infinity of possible stars but that we were in fact seeing different types of star which could be classified according to mass and which appeared to us at different evolutionary stages.

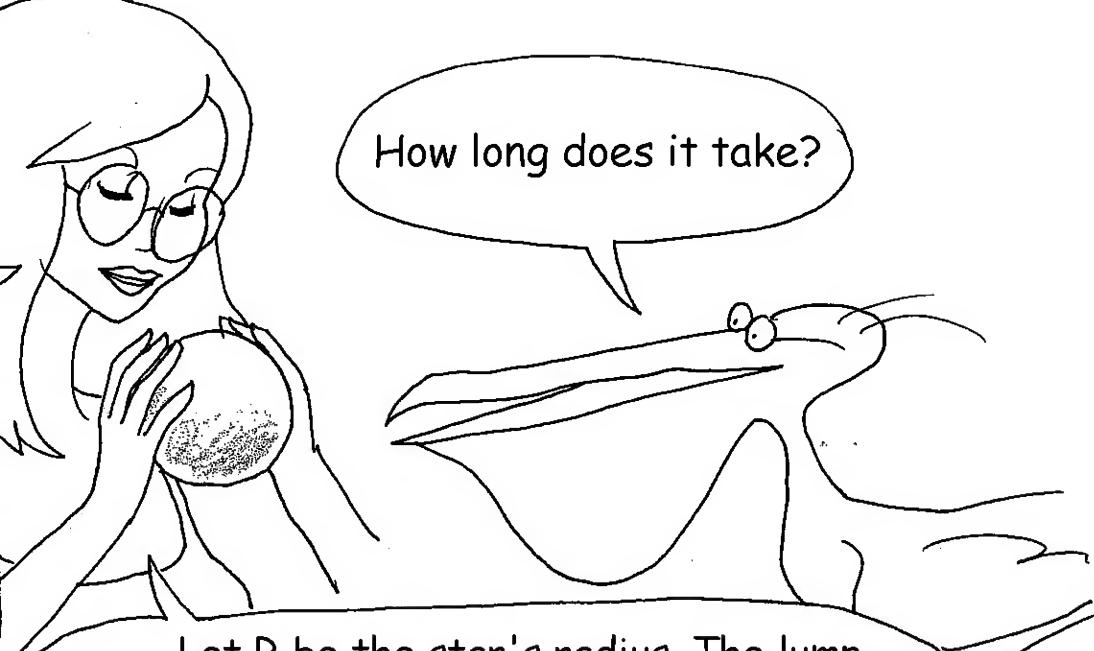


Ah, those massive stars, they burn their hydrogen at both ends

Stars form in gas clouds. Later we will see why and how "lumps" form: PROTO STARS. When FUSION begins, when the star starts to burn its "fuel", hydrogen. The greater the star's mass, the faster it will "burn" and the shorter its existence.

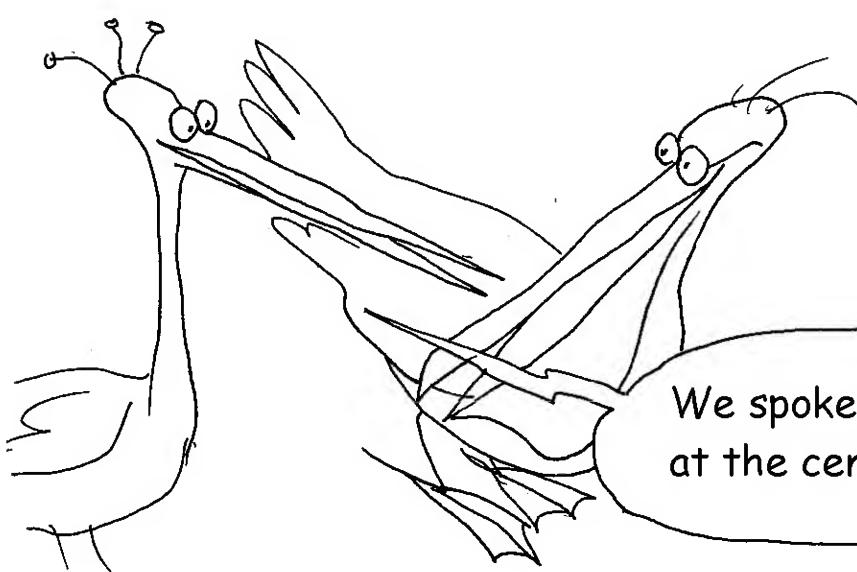
Jupiter is a "failed star" which radiates and contracts, but never light up. When there is sufficient mass, say ten times that of Jupiter, the star experiences a period of latency before the fusion reactions start.

How long does it take?



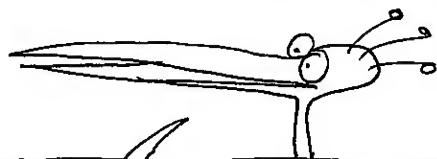
Let  $R$  be the star's radius. The lump contracts until its temperature reaches  $3000^\circ$ . The lump then ionises and pressure forces oppose the continuation of contraction. The amount of heat that needs to be evacuated, by radiation, is as the volume of the star, as the cube of its radius - the "radiator" is its surface  $4\pi R^2$ . The time taken for the dissipation of this heat, which allows contraction to begin again and results in fusion, varies therefore as the cube root of the star's mass, as its radius  $R$ .

We spoke of spheroidal agglomerations with negative mass situated at the centre of these great voids. How do these objects evolve?

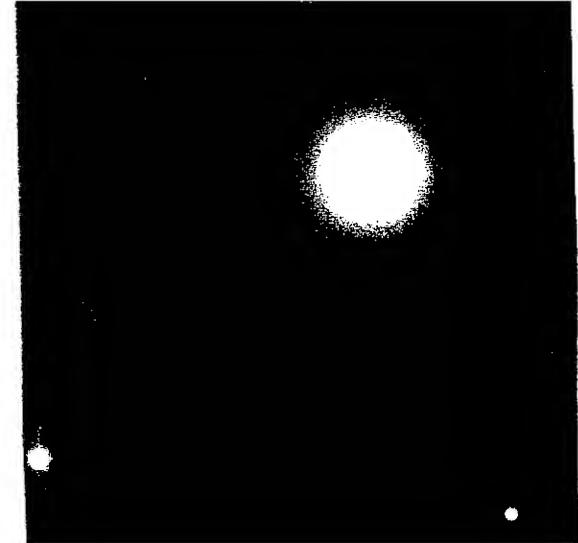




You'd need to be made of negative mass to be able to see these enormous proto-stars, radiating in the red and infrared, whose contraction time exceeds the Age of the Universe. Which means that they'll never light up !



So if I understand correctly, in this negaworld there are no real stars, no fusion, so therefore no planets and no LIFE ?

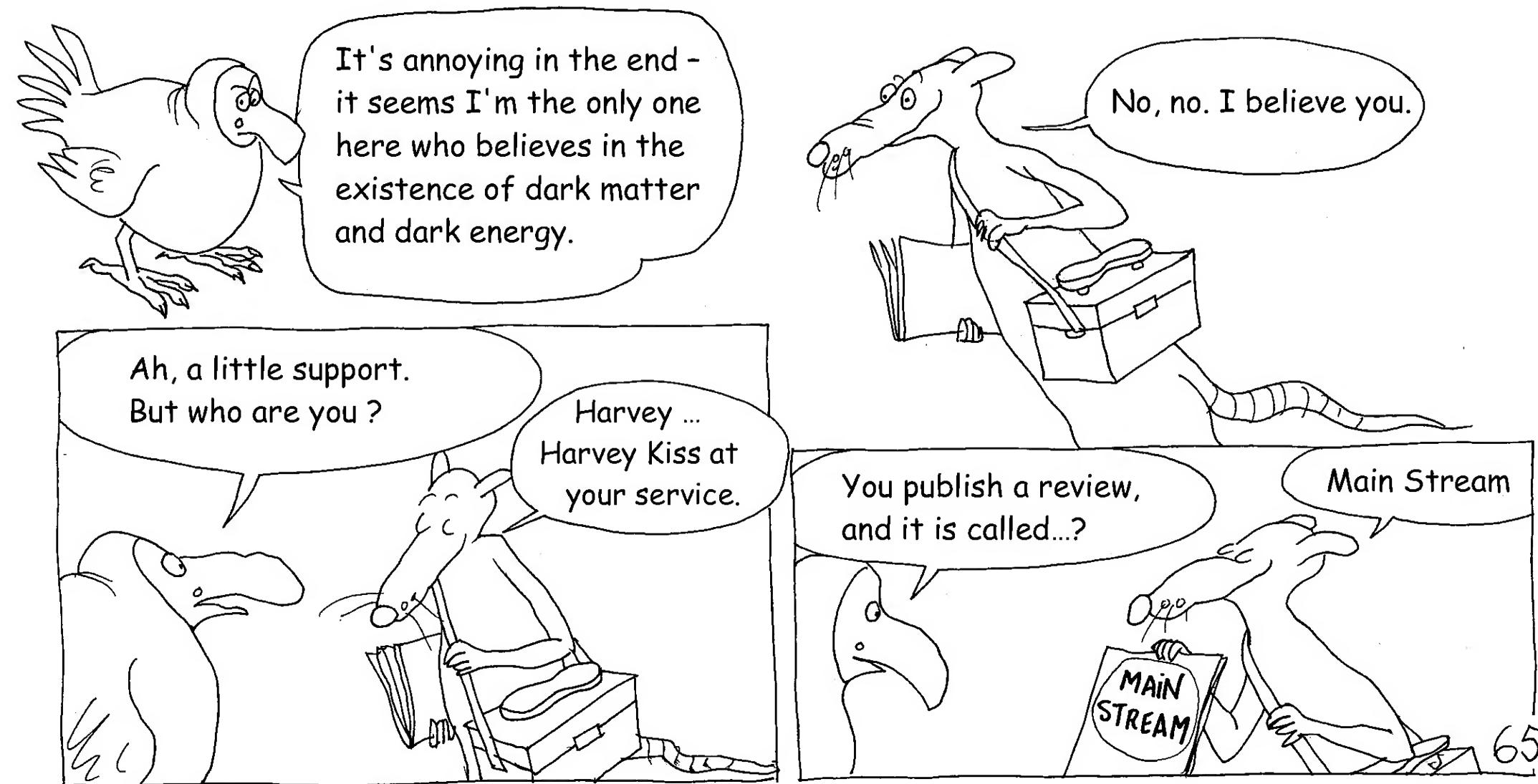


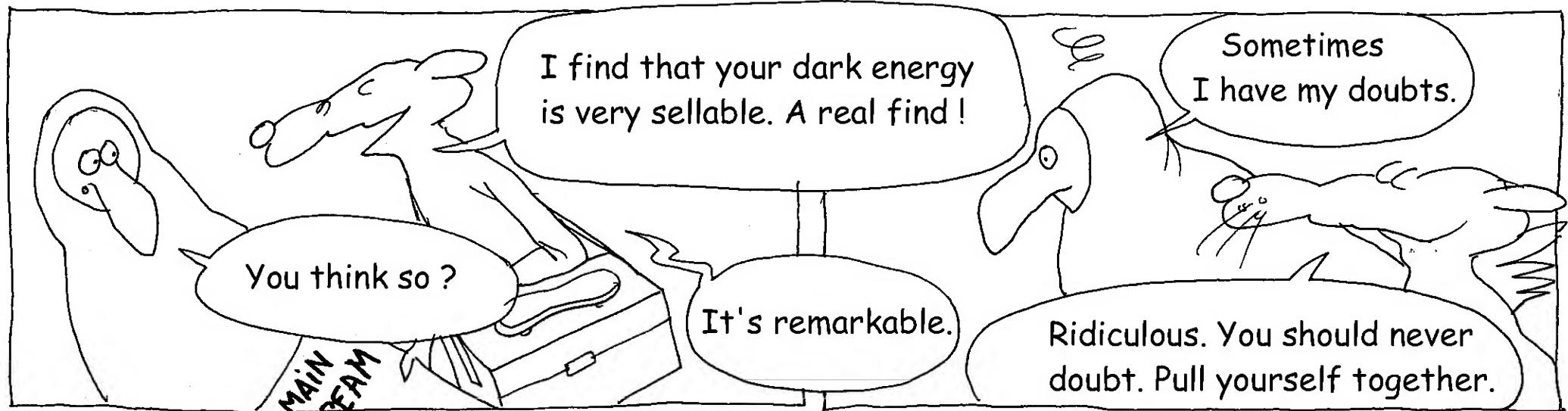
These objects are only the framework of our universe of positive mass.



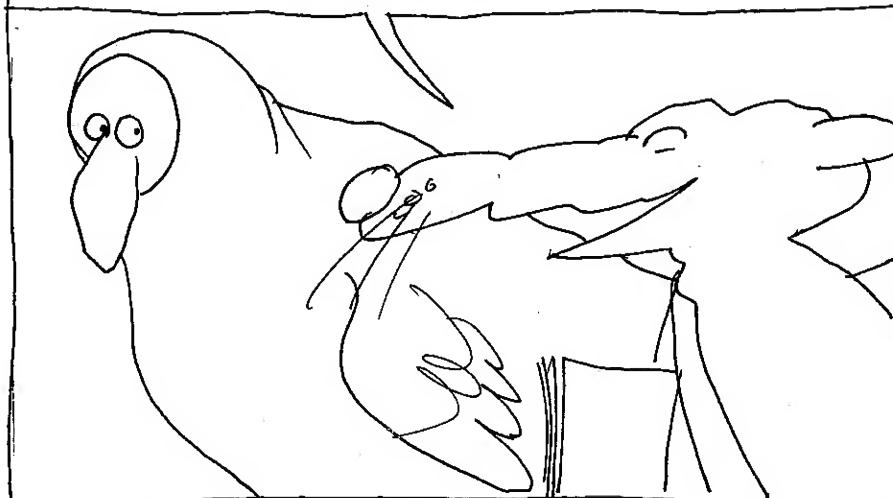
Ridiculous, phantasmagoric ! You can invent this sort of thing until the cows come home but DARK MATTER and DARK ENERGY, they are real !

# THE PROBLEM OF GALAXY FORMATION



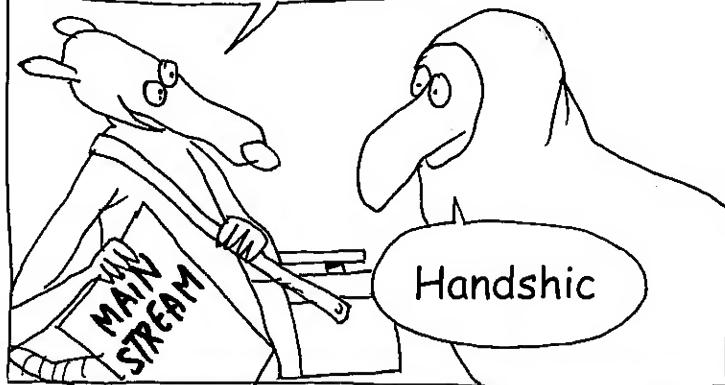


The most important thing is finding the words, fine words; black holes, dark matter, dark energy - the dark side is a great seller, believe me.



That is the only way to progress in science sir.

We are looking for an article on the formation of galaxies - What is your opinion on the question?

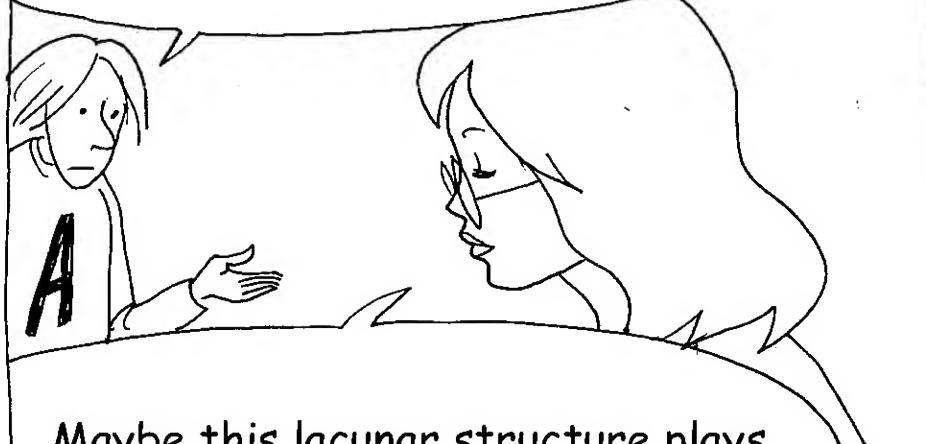


Tsss...science, it's cooking - you put in a packet of cosmic strings, a few magnetic monopoles, cold or warm dark matter, and maybe, to spice the whole thing up, a few mini black holes. No?

You think so?

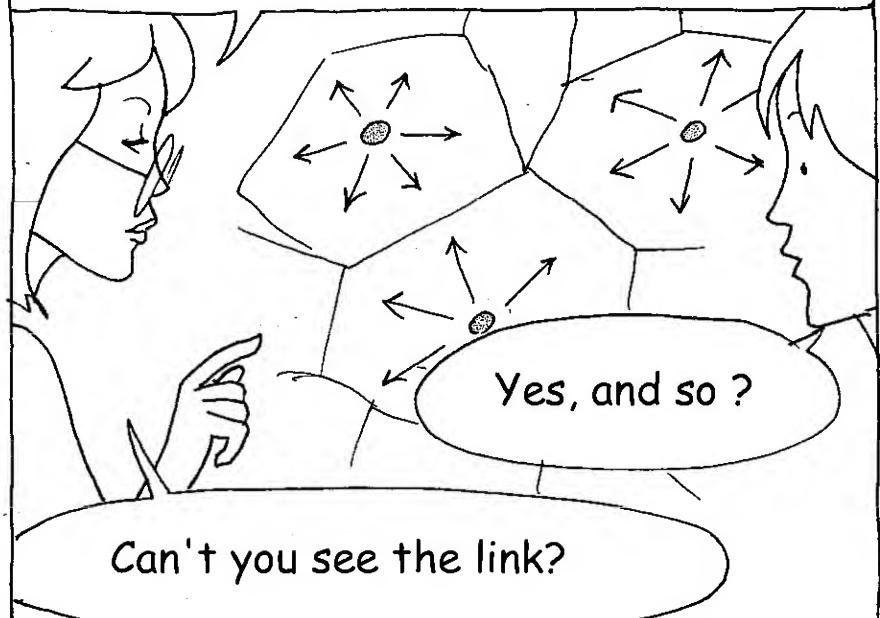
You write and me, I publish.

What do you think Sophie?



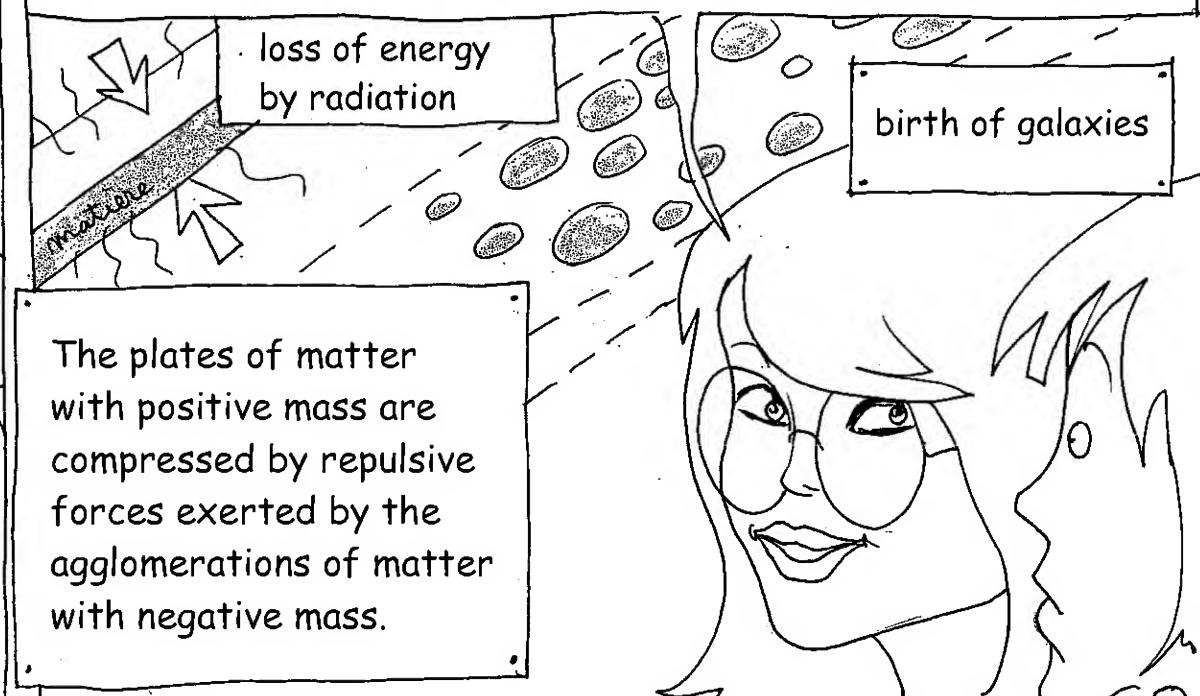
Maybe this lacunar structure plays a role in galaxy formation.

When we start from a mix of positive masses and negative mases, with a large superiority of the second over the first, this forms agglomerations through gravitational instability. In doing so matter of positive mass, ours, is repulsed into the residual space. But this happens quite violently and the matter, in the form of hydrogen and helium, is compressed as PLATES (\*)



(\*)

While matter with negative mass assembles in the form of spheres and so is unable to evacuate heat by radiation, a PLATE CONFIGURATION, however, represents the optimal radiator for matter, which can then cool by radiation after a strong temperature excursion. This destabilises the gas and the sudden cooling sets off gravitational instability and the formation of galaxies, ALL AT THE SAME TIME. That's why we never find young galaxies.

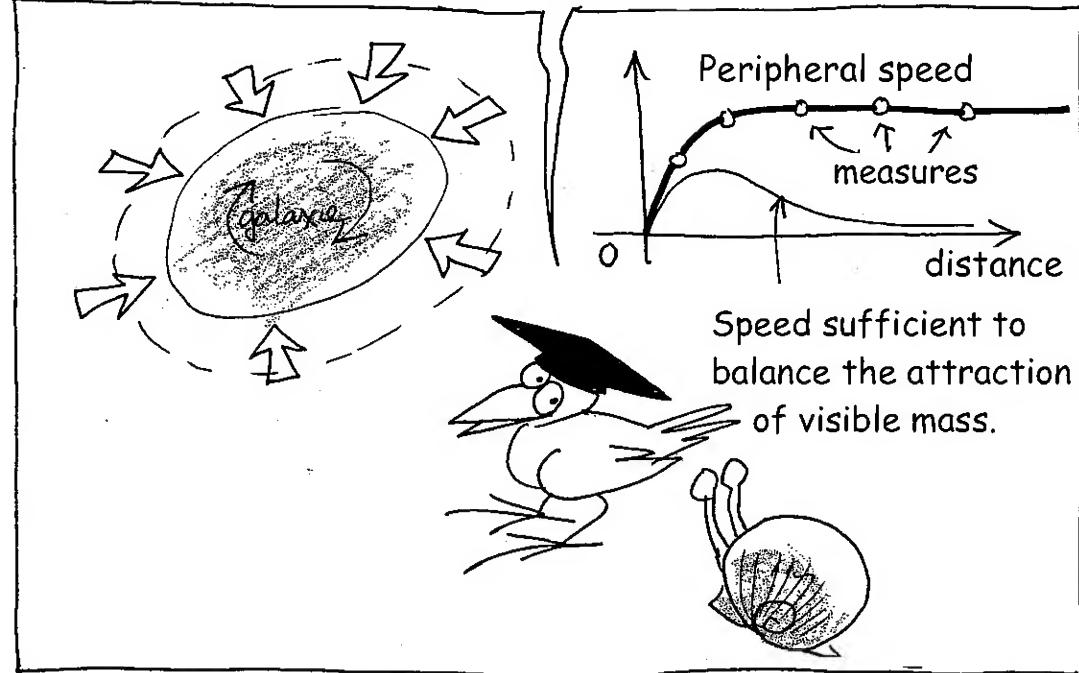


# GALAXY CONFINEMENT

Currently galaxies are distant from each other, like peas a metre apart. But at the time of their birth the young galaxies were close, like grapes in a bunch. They formed a **COLLISIONAL SYSTEM** and it was their interactions that gave them their rotation movements (\*). Then, as expansion separated them, collisions, while still exciting, became far more rare.

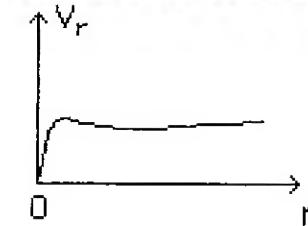
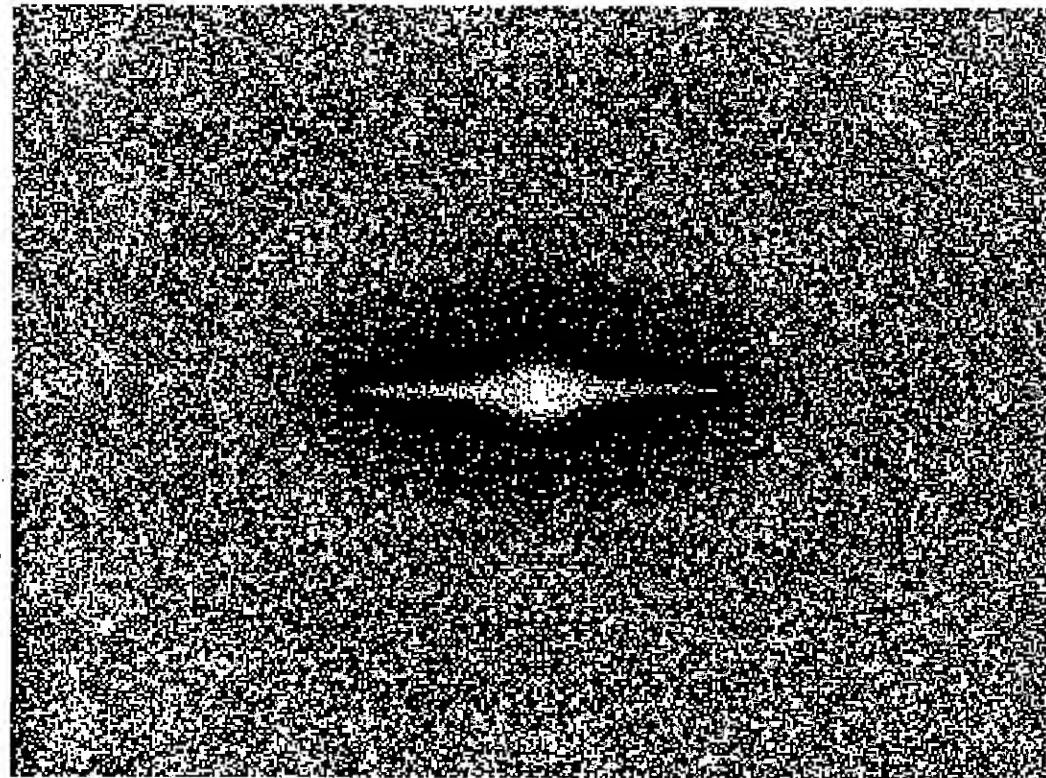


Matter with a negative mass isn't concentrated in agglomerations. It forms a gaseous ambience which exerts a **COUNTER-PRESSURE** on our own matter and, infiltrating between galaxies, it **CONFINES** them. Its presence at the frontiers of galaxies explains the peripheral superspeeds measured in interstellar gas.



(\*) In a gas collisions send molecules into rotation.

OK. Let's try to summarise this whirlpool of new ideas, which are totally different from those of the MAINSTREAM. If I understand it correctly, for you dark matter and dark energy are nonsense. Matter with negative mass is sufficient to explain everything. Its agglomerations fix and stabilise the LARGE SCALE LACUNAR STRUCTURE OF THE UNIVERSE, in the fashion of "nails". That gives an original plan for galaxy formation. The negative matter, by infiltrating between them, ensures their CONFINEMENT. It's as if they are nestled in the holes of Gruyère cheese.



The result of digital simulations (1992). At the bottom, the rotation curve deduced from that and which agrees perfectly with observation.

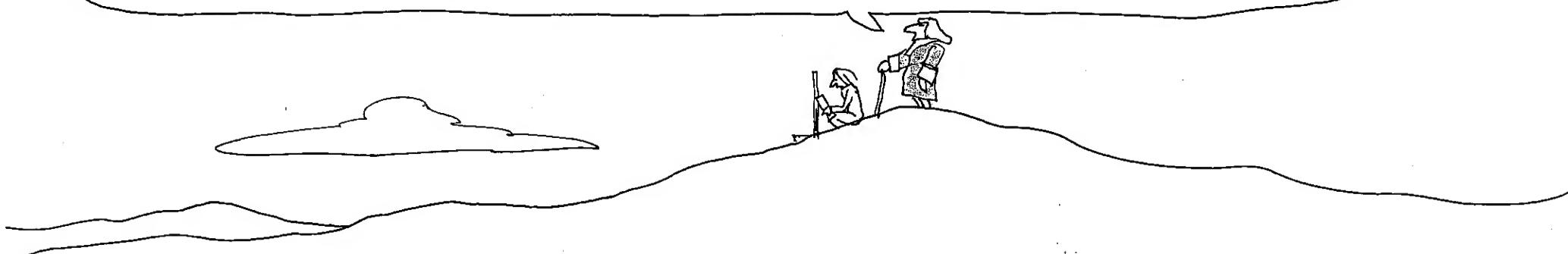
Just as the mini effects of gravitational lensing allow the New Astronomers to map the dark matter in the Universe, so people like Albert Bosma, opposite, adapt dark matter distributions that allow the rotation curves to be established. In the absence of new theoretical models everything comes down to Newton's law and adjustment techniques in order to fit in with observation

$$F = \frac{Gmm'}{d^2}$$



During the 17th century Toricelli understood that it is ATMOSPHERIC PRESSURE that makes mercury rise in the barometer he invented. Otherwise scientists would still be measuring the ABHORRENCE OF A VACUUM.

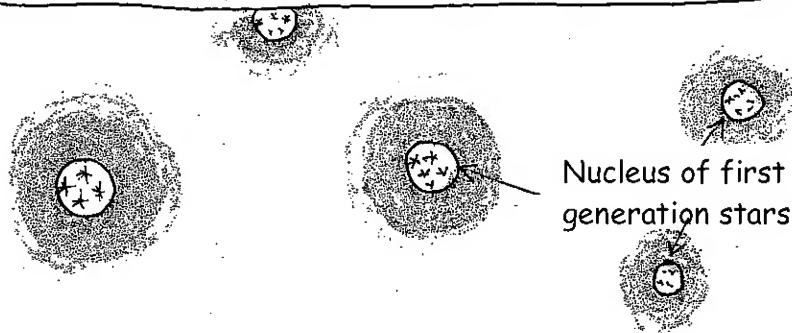
It's a great discovery = the abhorrence of a vacuum diminishes with altitude.



Why are light galaxies made of gas and massive ones not?



But as it is question of a galaxy ten times lighter, the heat communicated to the residual gas will be insufficient to allow it to escape from the gas. It will dilate therefore and form a sort of atmosphere. Young galaxies, still in very close proximity, will "rub against" each other which will make the gaseous auras rotate (but not the central nucleus, made of stars).



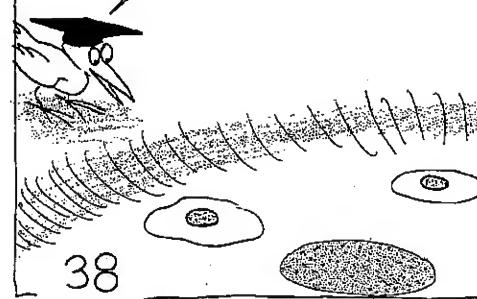
The "first generation" stars form immediately and carry the residual gas at a high ambient temperature. For massive galaxies this heating is so powerful that the thermal agitation velocity

$$V = \sqrt{\frac{3kT}{m}}$$

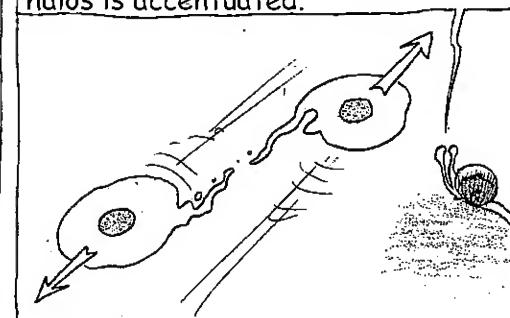
will surpass the LIBERATION VELOCITY (\*) of the galaxy - Therefore this gas will be lost in space and become so rarefied that collisions between atoms can only bring about its RADIATIVE COOLING.

As was already described in 1968 in a THOUSAND MILLION SUNS : page 38 :

Light galaxies have a "white" and a "yolk", while heavy galaxies, called ELLIPTIC, only have a large yolk.

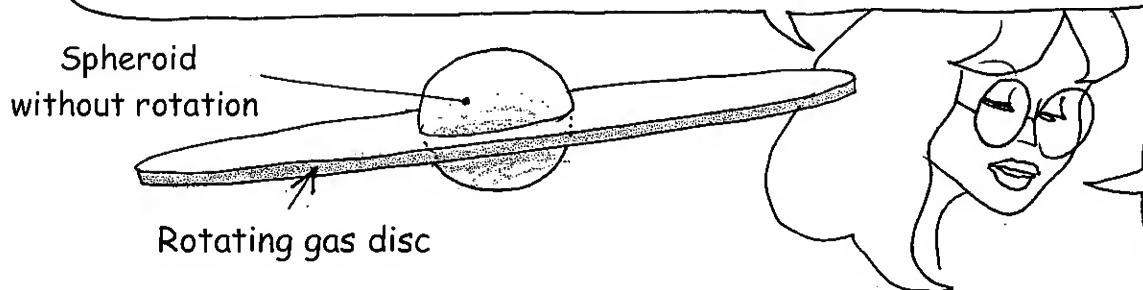


The residual gas halos of light galaxies increase the chances of interaction between objects. The rotational movement of the gas halos is accentuated.



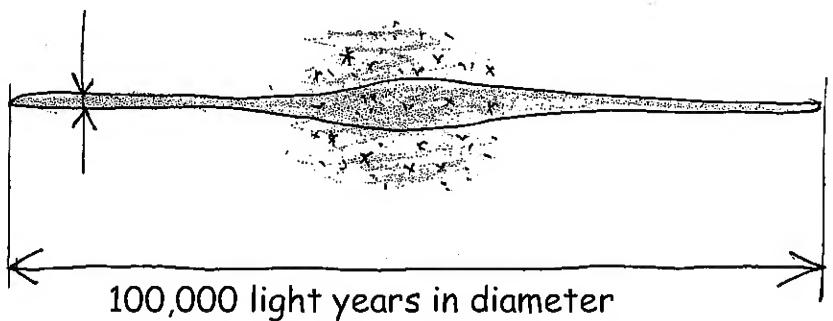
(\*) This liberation velocity is of the order of 1000 km/s. By applying  $\frac{1}{2}mV^2 = \frac{3}{2}kT$  (annex) we find that galaxies should be bathing in a gas at a temperature of tens of millions of degrees, as was shown.

Expansion distances galaxies from each other - The gaseous auras conserved by light galaxies, which form collisional groups of atoms, cool down by emitting radiation. Conserving the ANGULAR MOMENTUM it has acquired during its encounters, the gaseous mass mutates into a very flat disc, associated with the spheroid constituted of first generation stars which DO NOT ROTATE and which will give rise to hundreds of globular clusters, of 100,000 stars, constituting the "fossil galaxy".



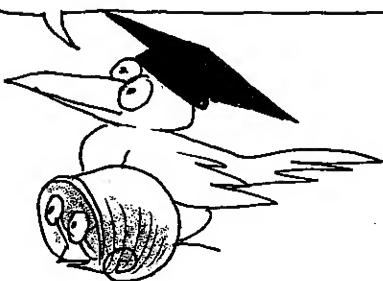
The radiative cooling destabilises the gaseous mass and brings about the birth of second generation stars through gravitational instability.

300 light years thick

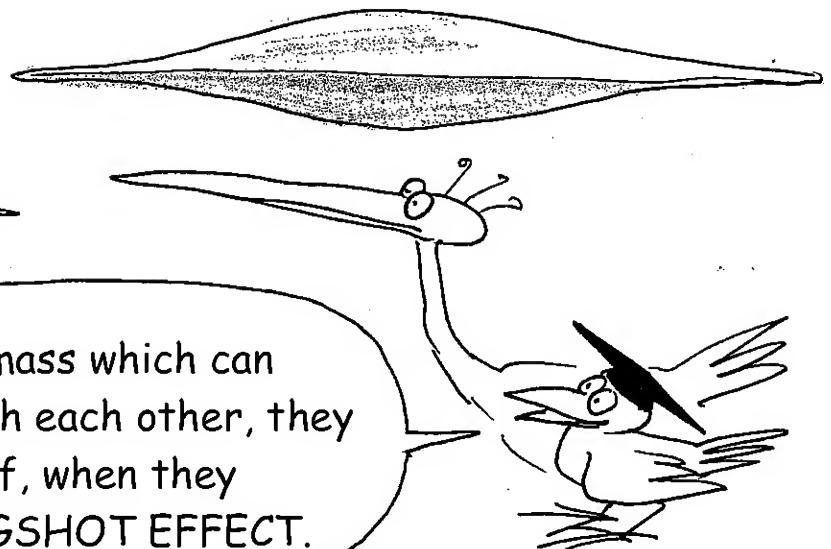


The thickness of the gas disc remains constant because the UV radiation emitted by the young stars reheats it and stops it from flattening completely. Nevertheless, we could compare the geometry of a galaxy that has it, to that of a CD-Rom.

In other words, these galaxies function like a flushing cistern. When the temperature of the gas drops, new stars are created which reheat it.



There's one thing I don't get : spiral galaxies, when we see them sideways on, don't seem that flat - and we can barely distinguish the frontiers between the two star populations, that of the halo and that of the disc.

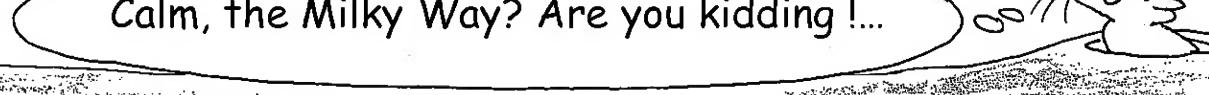


Interstellar gas is fragmented into clouds of very varied mass which can represent 100,000 solar masses. Stars do not interact with each other, they ignore each other's existence (\*) but they leave the disc if, when they encounter a stellar mass, they are accelerated by a SLINGSHOT EFFECT.

The interstellar milieu is as impermanent as a cumulus on a sunny day. Supernovae explosions (one per century, or a million for the entire galaxy) disperse them constantly over a radius of more than one hundred million light years, creating a disorder in the fashion of bangers exploding in an eiderdown. One storm passes, another will develop further away because of gravitational instability.



Calm, the Milky Way? Are you kidding!...

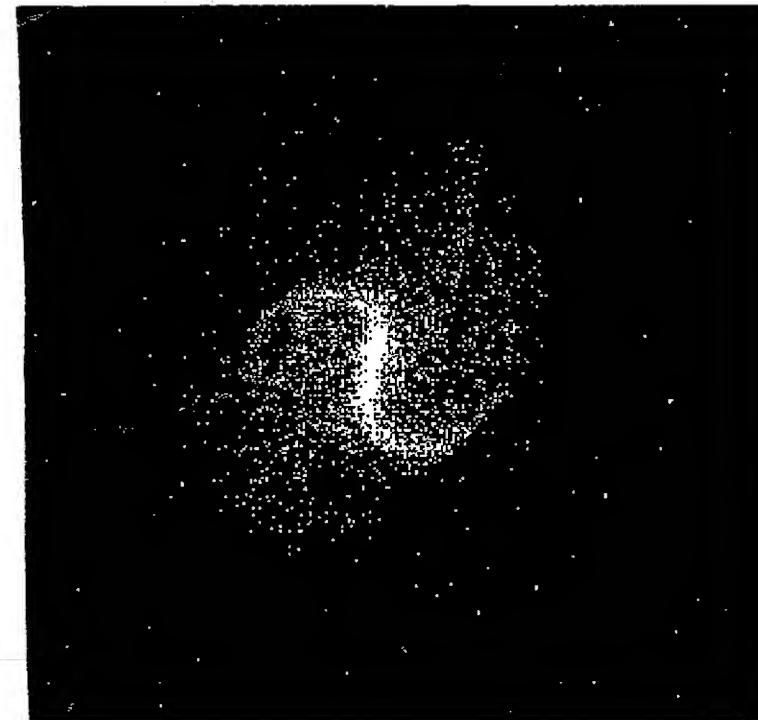


(\*) Close encounters between stars are about as frequent as would be those of two ants moving around the whole of France.

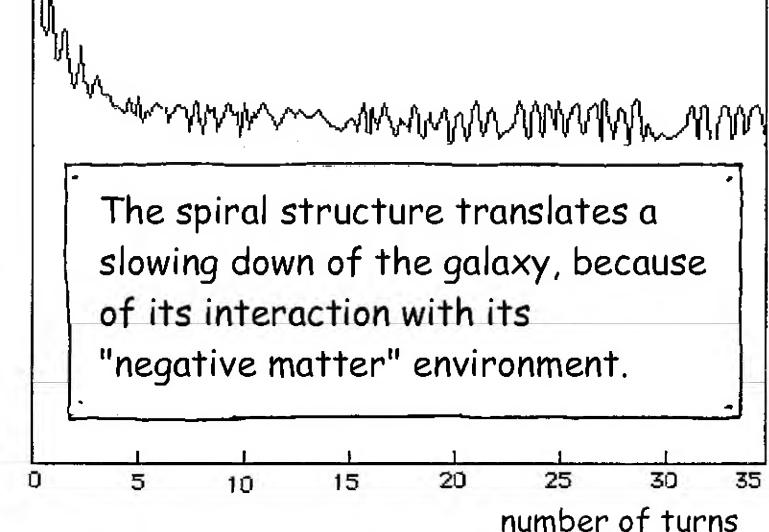
# SPIRAL STRUCTURE



In 2002, while making a lump of matter of positive mass interact, in rotation in a hole arranged in a distribution of negative masses, there was an immediate creation of a barred spiral, stable during 30 rotations. But let's abandon this theme of research in the face of the hostility of competitors.



Momentum of the positive population

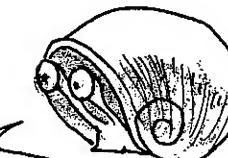


The spiral structure translates a slowing down of the galaxy, because of its interaction with its "negative matter" environment.

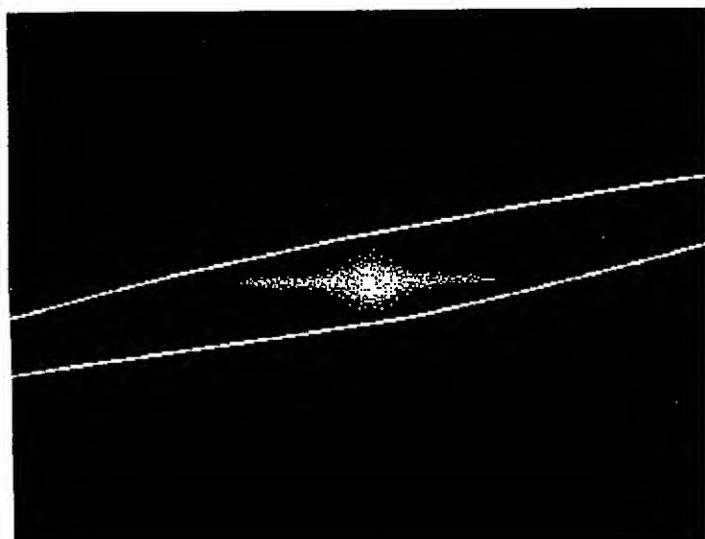
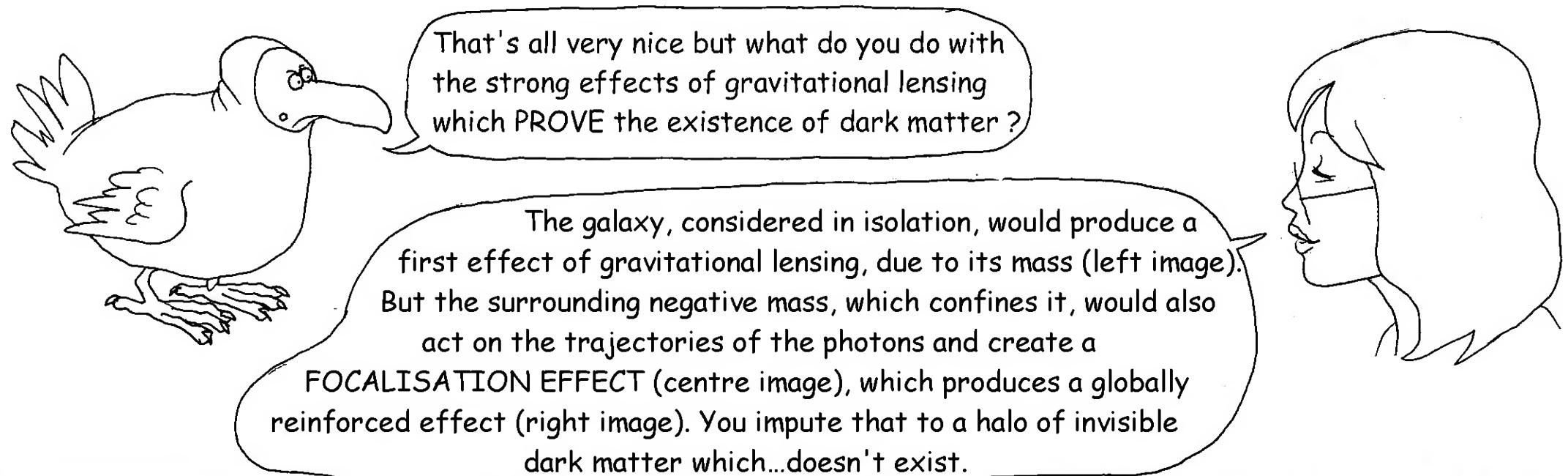
There the idea is very simple = the galaxy, confined in its Gruyère cheese "hole" and revolving inside it, is subjected to the effect of the DYNAMIC FRICTION phenomenon.



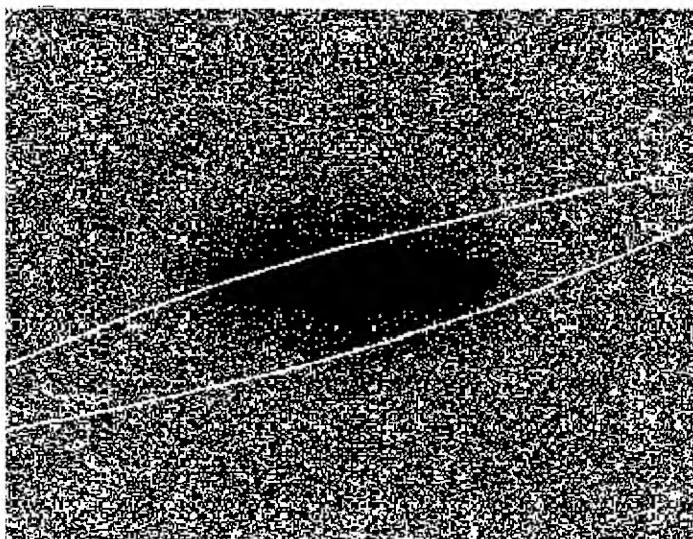
Like when we stir our cappuccino in a cup with a spoon.



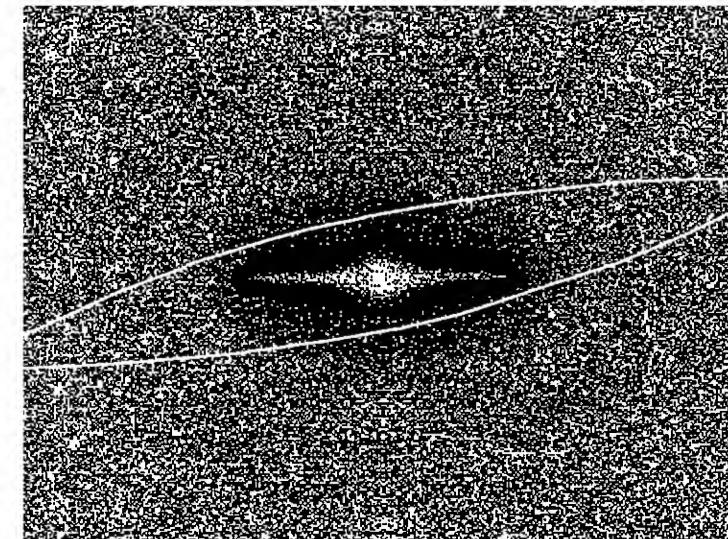
# MYTHICAL DARK MATTER



Gravitational lensing effect, lone galaxy

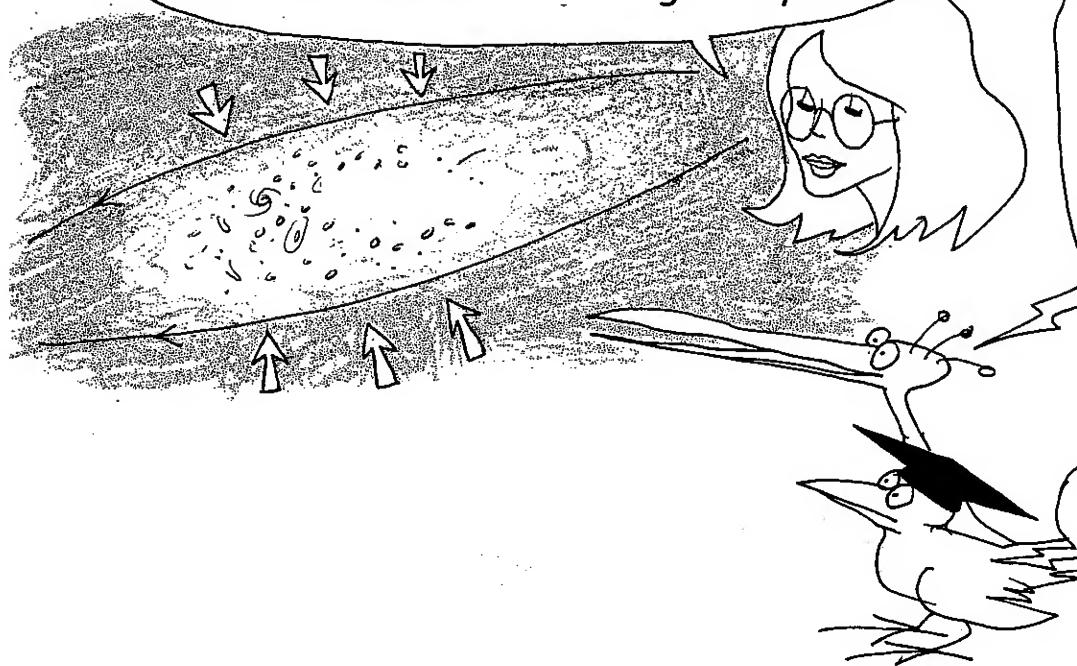


Focalisation due to the action of the negative mass



The two effects combined

For GALAXY CLUSTERS, the same cause, the same effects = reinforcing of the focalisation of light rays.

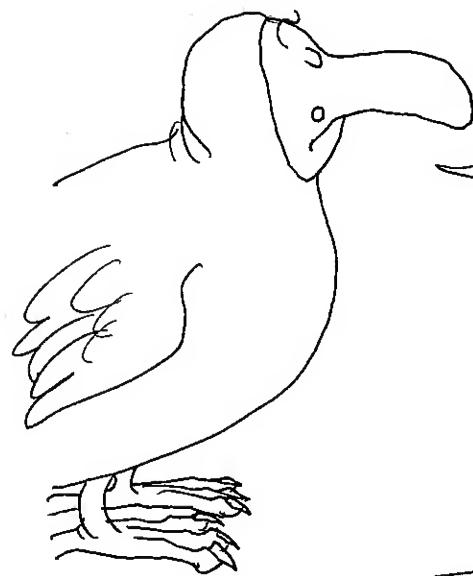


If I understand it correctly, this matter with negative mass exercises a counter-pressure at different levels. First it ensures a perennity for the large-scale structure of the lacunar Universe - Then it maintains the galaxies in the cluster - On a lesser scale it confines the galaxies. But couldn't it infiltrate into the interior of galaxies ?

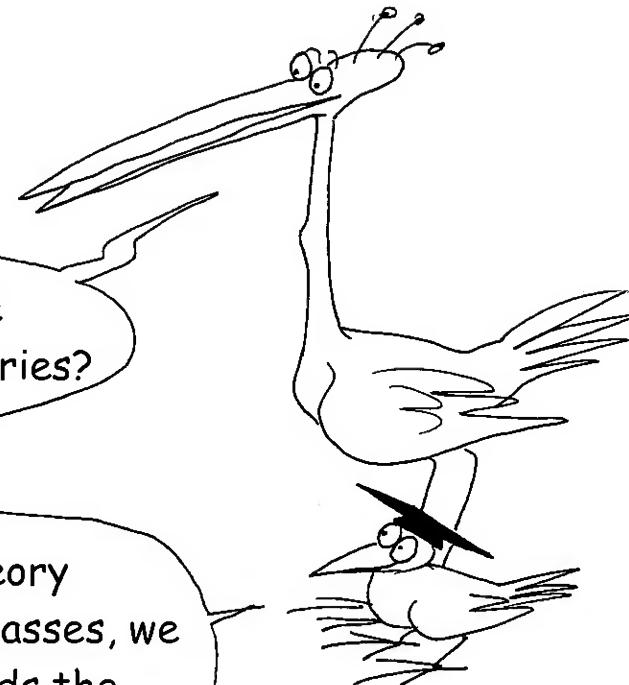
Yes, and we find it, with very low densities between stars.

That's funny ! At very large scale, it's matter which is structured like a Gruyère cheese, the agglomerations of negative mass are localised at the centre of the "holes". At lesser scales it's the contrary. It's the negative mass matter that becomes lacunar. Galaxies, and on a smaller scale stars, lodge in the "holes".





OK...you've found an ALTERNATIVE interpretation of this phenomenon. Personally, I prefer the one based on DARK MATTER.



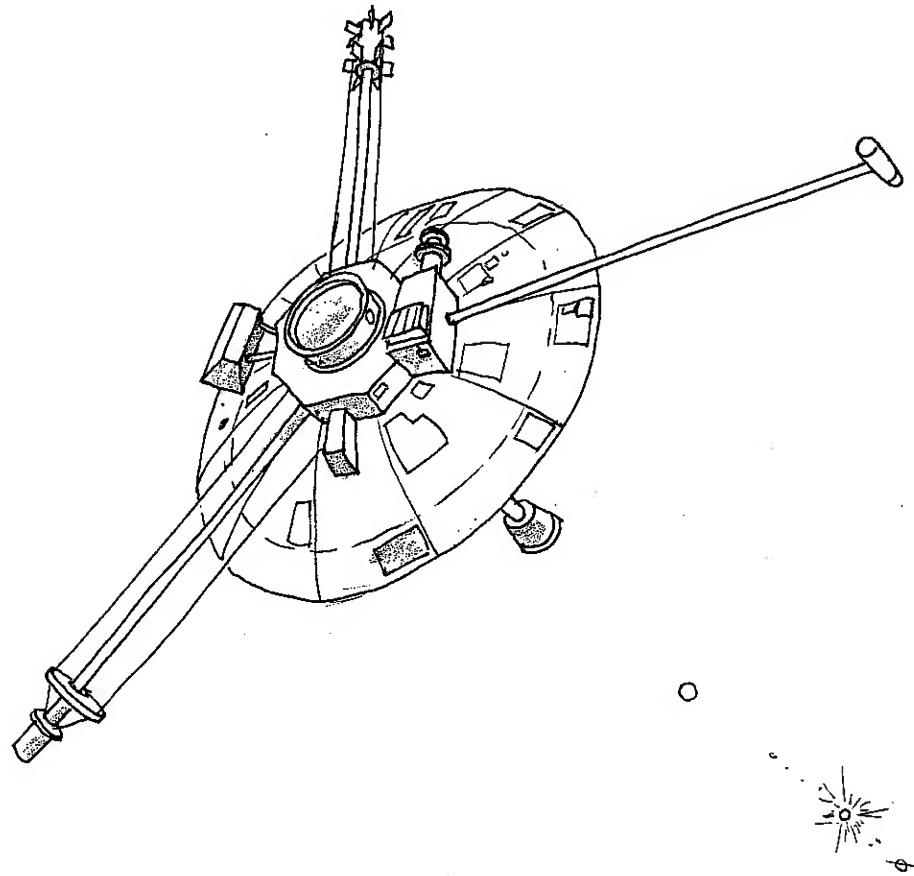
Do you mean that it would be impossible to opt for one or the other of these theories?

It should be remembered that with this theory where we have two matters with opposing masses, we kill two birds with one stone because it avoids the need for another ingredient = DARK ENERGY.



The ideal would be to consider an observation that could be accounted for by the negative mass and not dark matter.

# THE PIONEER EFFECT



In 1972-73 NASA launched two identical probes: PIONEER X and PIONEER XI. Benefiting from the SLING EFFECT as it passed Jupiter, they were able to attain a speed that allowed them to escape the Sun's attraction awns leave the solar system in (???). Fuelled by a nuclear generator they were able to send back signals until (???) and an unusual phenomenon was noticed. The probes were subject to a deceleration, extremely small but perfectly measurable (\*). Everything was tried to account for this phenomenon including the fact that the solar system, near the Sun, holds a certain quantity of DARK MATTER.

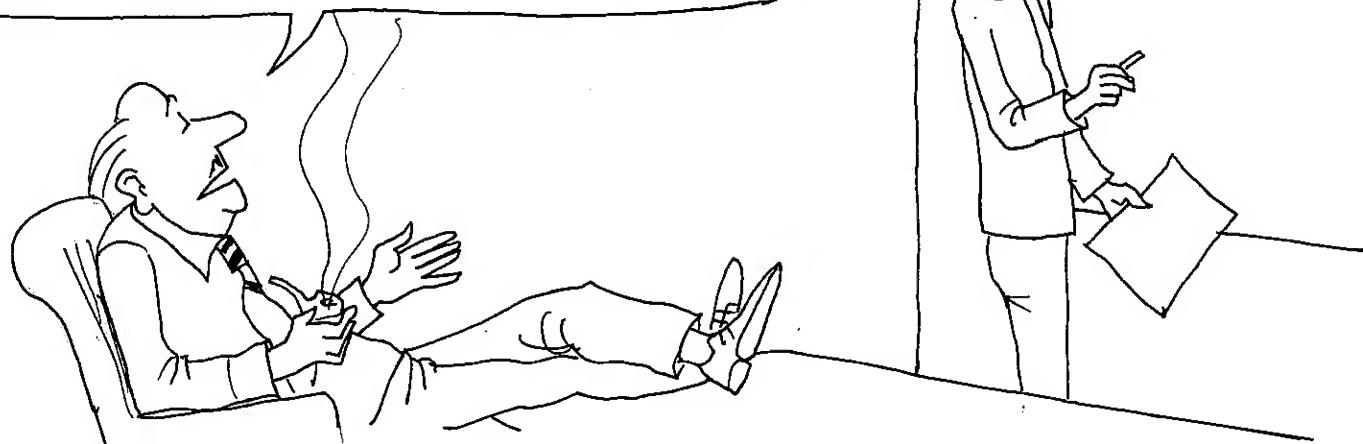
But for the first time the cover-all explanation didn't work...



The solar system functions like precision mechanics, governed by Newton's law. As time has passed, the computer has allowed us to position the near planets to within 20 metres at any moment. Such precision precludes all modification of the central mass, which governs the movement of planets with a mass of more than a hundred thousandth that of the Sun. However, in order to be able to account for the acceleration observed, the quantity of dark matter to be added to the classic model would largely exceed this value. We are obliged to look elsewhere for the cause of this phenomenon therefore. Currently (2008) the effort is being put into a (empirical) modification of Newton's Law (MOND or Modified Newton Dynamics) (\*). This brings about a re-examination of the fundamental principles of General Relativity. But, beyond that, the adjustments required to make this distant deceleration appear no longer fit in with the low distance dynamic of the Sun (telluric planets).

I am willing to recognise that your modified Newton's Law accounts for the deceleration of the probes, but if I use your law to send a probe to Mars I'll miss my target and by quite a way. The dates of solar and lunar eclipses will no longer fit in with the EPHEMERIDS. What should we do ?

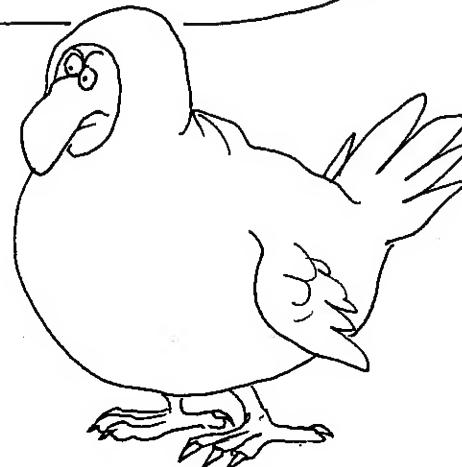
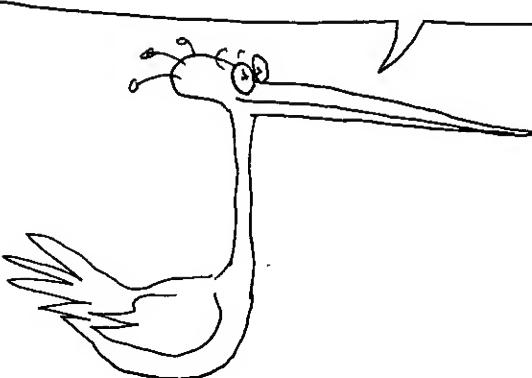
$$F = \frac{GMm}{d^2} - \frac{\alpha}{d^3}$$



(\*) All other causes of physical or technical character have been registered and eliminated.

Therefore the "Dark Matter" hypothesis is not able to resolve the unavoidable enigma brought up by this INCONTESTABLE phenomenon and shown by the space probes PIONEER X and PIONEER XI.

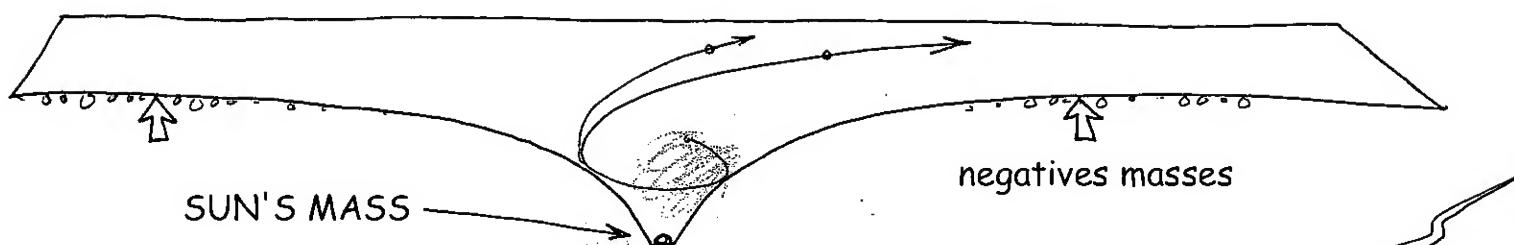
All that remains is to impute this phenomenon to the REPULSIVE action of the tiny quantity of negative mass present near the sun.



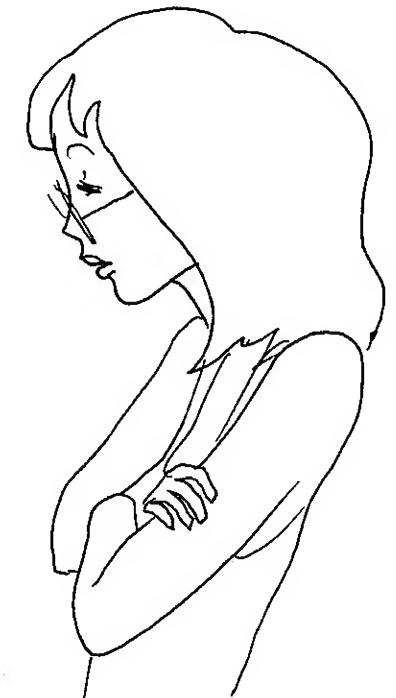
So what's it about then.



The "ping pong balls" then lift up very slightly, from a distance, the surface on which the probes move. The "uphill side" is simply slightly more steep.



It's the ONLY explanation that has any sense at all.



# Bi-METRIC UNiVERSE

see my papers :

J.P.Petit : The missing mass problem. *Il Nuovo Cimento B*, July **1994** Vol 109, pp. 697-708

J.P.Petit : Twin Universe Cosmology. *Astronomy and Sp. Sc.* **1995** , 226 , pp. 273-307

Bigravity as an interpretation of cosmic acceleration. J.P.Petit & G.D'Agostini Dec. **2007** <http://arxiv.org/abs/0712.0067>

Bigravity : A bimetric model of the universe. Exact nonlinear solutions. Positive and negative gravitational lensings

J.P.Petit & G.D'Agostini **Jan.10 , 2008** <http://arxiv.org/abs/0801.1477>

Bigravity : A bimetric model of the Universe with variable constants, including the speed of light. J.P.Petit & G. D'Agostini

**May 9th 2008** <http://arxiv.org/abs/0803.1362>

Five-dimensional bigravity : New topological description of the Universe. J.P.Petit & G. D'Agostini

**May 9 th 2008** <http://arxiv.org/abs/0805.1423>

And :

## A Bi-Metric Theory with Exchange Symmetry

S. Hossenfelder\*

Physical Review July 2008

*Perimeter Institute for Theoretical Physics*

*31 Caroline St. N, Waterloo Ontario, N2L 2Y5, Canada*

(Dated: July 17, 2008)

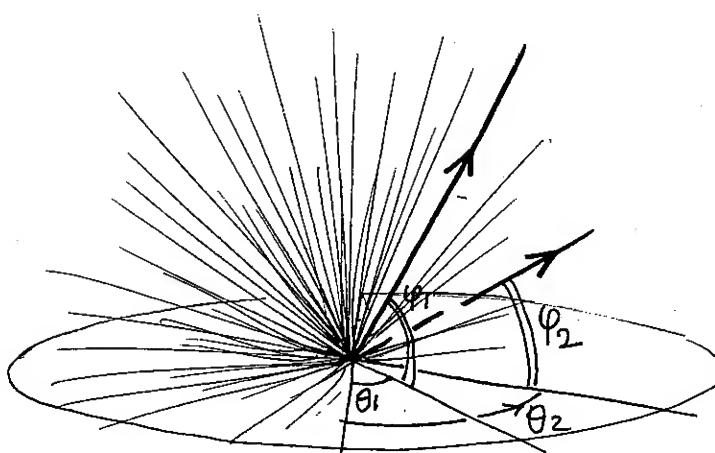
We propose an extension of General Relativity with two different metrics. To each metric we define a Levi-Cevita connection and a curvature tensor. We then consider two types of fields, each of which moves according to one of the metrics and its connection. To obtain the field equations for the second metric we impose an exchange symmetry on the action. As a consequence of this ansatz, additional source terms for Einstein's field equations are generated. We discuss the properties of these additional fields, and consider the examples of the Schwarzschild solution, and the Friedmann-Robertson-Walker metric.

The world of Science is paved with stories like this.  
This parenthesis closed, let's continue...

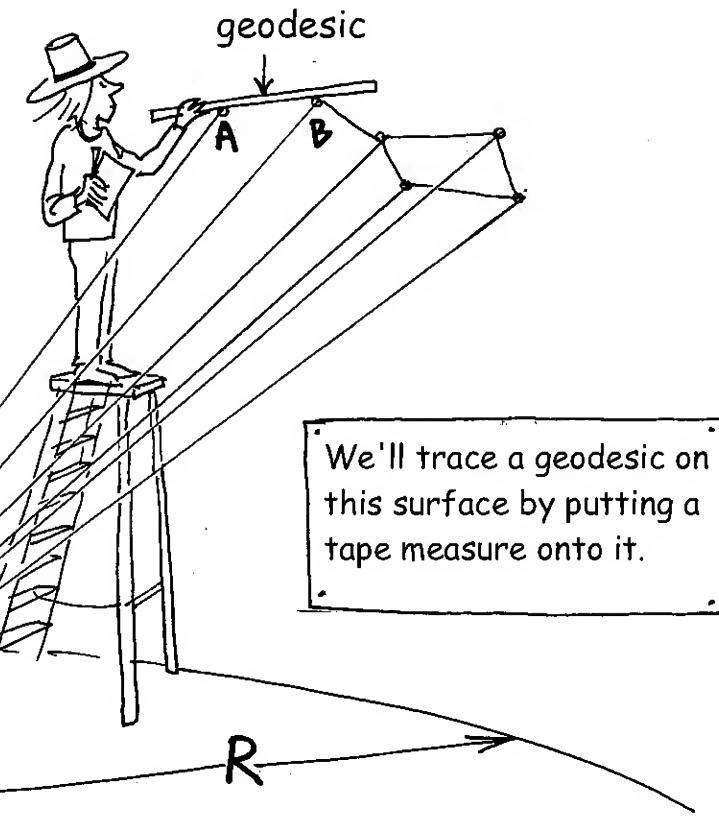


# THE MYTH OF THE CAVE

During the 4th century BC the Greek philosopher Plato developed the idea in which the perception that man could have of the world was comparable with the observation of dancing shadows projected from outside onto the wall of the cave where he lived, shut in and unaware of the true nature of the things surrounding him. With the coming of the Theory of Relativity, the myth returned. In effect, we have said that the revolution of this beginning of the century consisted of restituting the phenomena on a SPACE-TIME HYPERSURFACE. We are going to introduce an image. Everyone knows those lights made up of a bunch of fibre-optics strands, which just point in a direction which can be described with two ANGLES, the azimuth  $\theta$  and the site  $\varphi$ . It's an image of a PRE-METRIC SPACE where the concept of distance is void of all meaning as two fibres are only separated by ANGULAR DIVERGENCES.

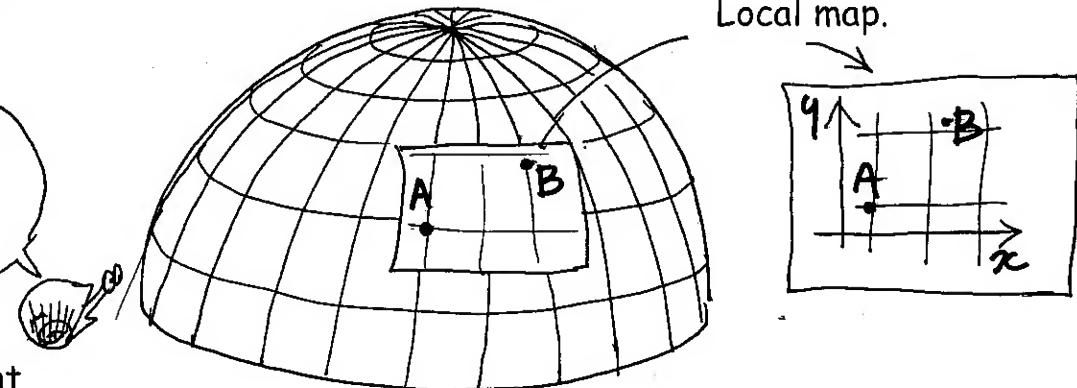


Imagine a bundle of such fibre-optics, infinitely tight. Some emit light, others not. By projecting these lights, eventually of different colours, onto a spherical screen we would be making a classic PLANETARIUM. We could also measure the DISTANCE SEPARATING TWO OF THE IMAGES on the screen by using a GEODESIC.



The length of the geodesic arc  $\widehat{AB}$  will be proportional to the radius  $R$  of the spherical screen of our planetarium. We will call this magnitude  $R$  "space scale factor" or "gauge" (\*)

After that we can MAP the screen with a grid using two families of curves that we'll call COORDINATES.



(\*) The terminologies used vary according to different authors : space scale factor, gauge, warp, factor etc.

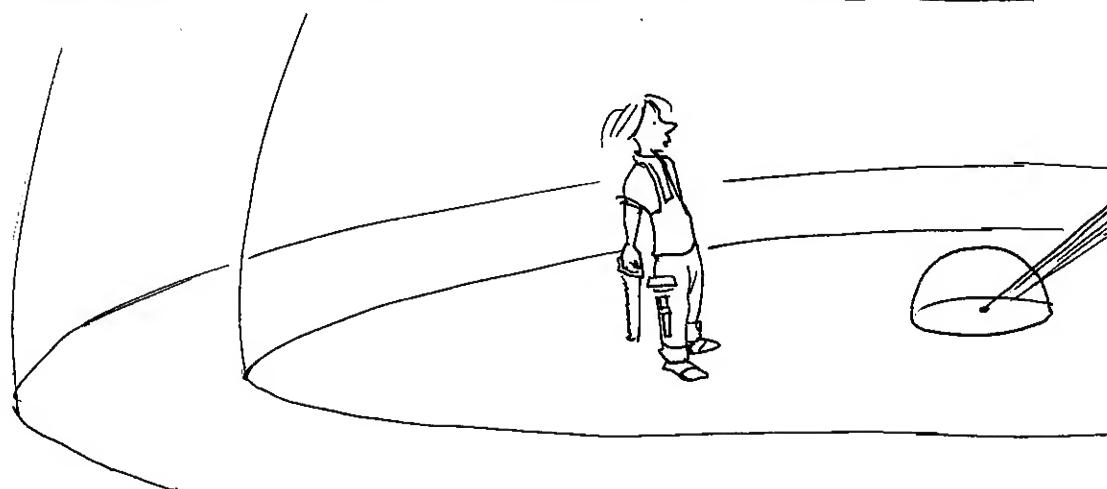
# BIMETRIC

It is clear that the true way to establish a POSITION in this space isn't the group of two lengths ( $x, y$ ) but the angles ( $\theta, \varphi$ ). This will be even more true if the screen...inflates, if our planetarium is in expansion. Then, for example "being immobile" in relation to this "space", will translate as =

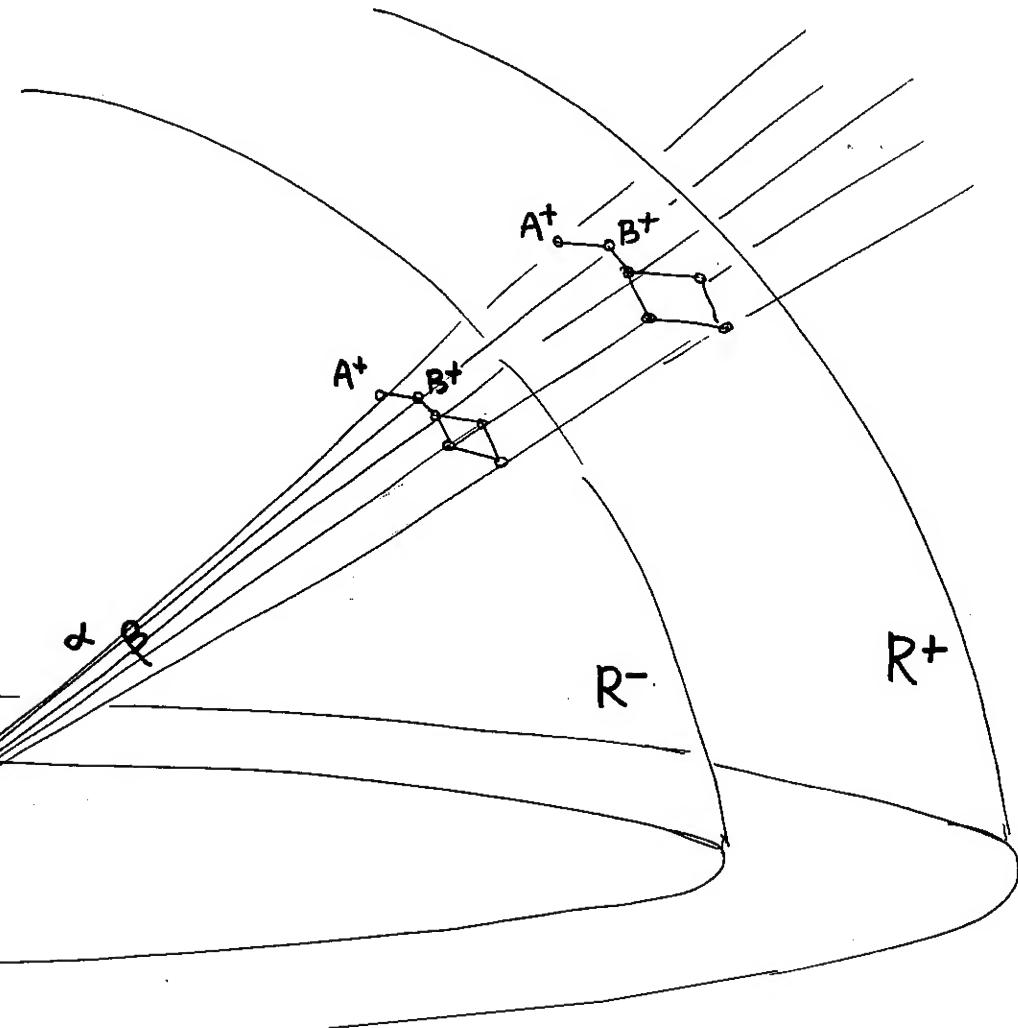
$$\theta = \text{constant}$$

$$\varphi = \text{constant}$$

We would then say that we are COMOBILE in relation to this space.



Let us now imagine that this ensemble of "positions", of "places" ( $\theta, \varphi$ ) can be projected not onto ONE screen but onto TWO.



We would have two different ways of MEASURING the distance separating the points  $\overbrace{A^+ B^+}$   $\overbrace{A^- B^-}$  of the image of the same "light rays"  $\alpha$  and  $\beta$  according to the screen chosen.

# (PLATO)<sup>2</sup> or THE TWIN UNIVERSE

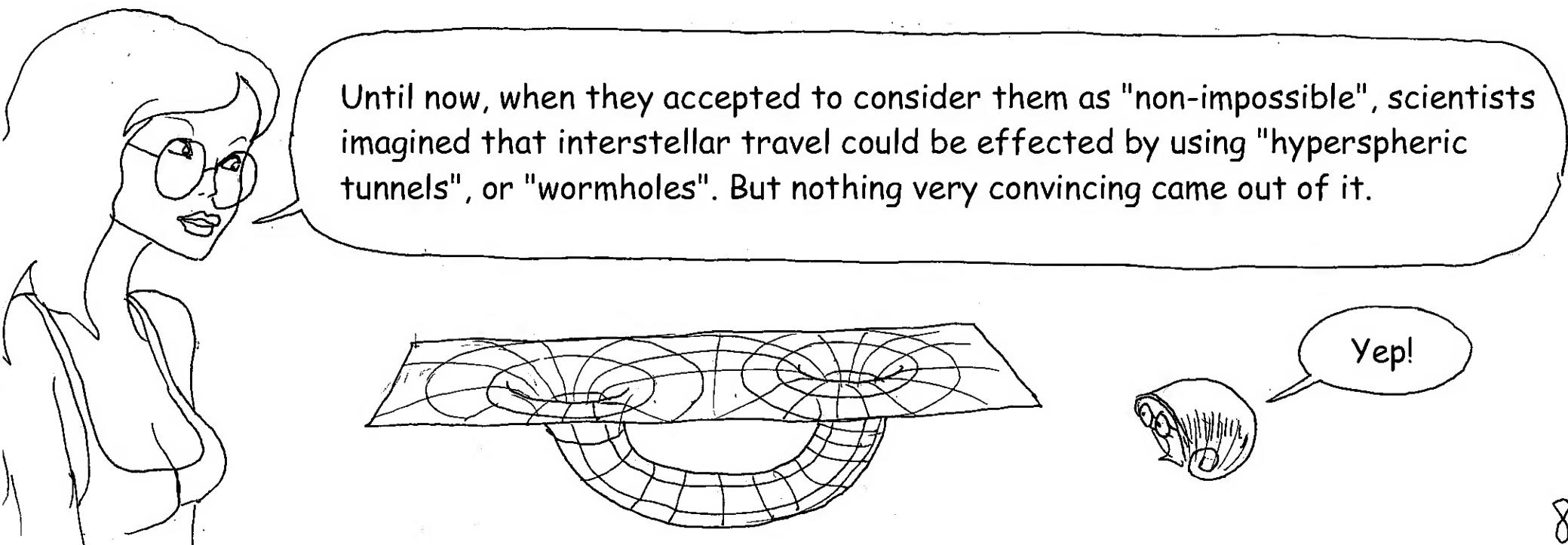
This BIMETRIC conception of the Universe represents a PARADIGM CHANGE extremely difficult to conceive of. It means accepting Plato to the letter, with a NON-METRIC underlying structure, where the different places  $\alpha$  and  $\beta$  (the "fibre optics") are located with the help of ANGLES  $(\theta_\alpha, \varphi_\alpha)$  and  $(\theta_\beta, \varphi_\beta)$  - This "projection system" (the planetarium) is projected onto two surfaces (leaves, branches, anything) whose scale factors  $R^+$  and  $R^-$  can be very different, including "from one place to another". For a mathematician surveyor it is something completely "natural" to give an underlying structure, where the positions are located using angles called MANIFOLDS, several sheets whose WARP FACTORS  $R^+$  and  $R^-$  can be totally different - if these 4D HYPERSURFACES are MINKOWSKI SPACES the objects cannot move faster than the speed of light of the space under consideration. But these velocities can be very different (for example  $C^- \gg C^+$ ) - of course we envisage that objects of mass  $m^+$  (previously called  $m$ ), and the objects of mass  $m^-$  and energy  $E^-$  (previously designated by  $\bar{m}$  and  $\bar{E}$ ) taking the route  $\widehat{A^+B^+}$  and  $\widehat{A^-B^-}$  inscribed in different SHEETS or BRANES, that we could consider to be TWIN UNIVERSES  $U^+$  and  $U^-$ , which in fact constitute just one single TWIN UNIVERSE  $U$ . This SECOND UNIVERSE isn't elsewhere, just as the particles with negative energy aren't ELSEWHERE. Objects of opposed mass and energy are immersed in the same universe and where

THEY CAN ONLY INTERACT THROUGH GRAVITATION

INTERSTELLAR TRAVEL would therefore be non-impossible and could be effected by using the "corridors" of a TWIN UNIVERSE having a higher luminic speed  $C^-$ . A vehicle whose mass was inverted would be :

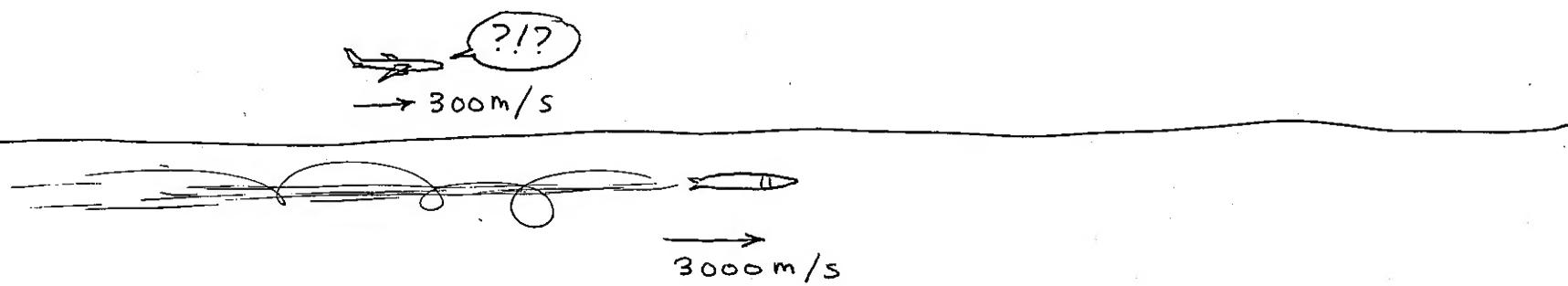
- invisible
- repulsed by the mass of Earth

By alternating its presence through a phenomenon of a quantum nature in the two twin universes, it will fall in one of these worlds and rise in the other, the rapid alternation of these two sequences giving an impression of immobility, so of ANTIGRAVITATION, to an observer made of positive mass.



# INTERSTELLAR TRAVEL

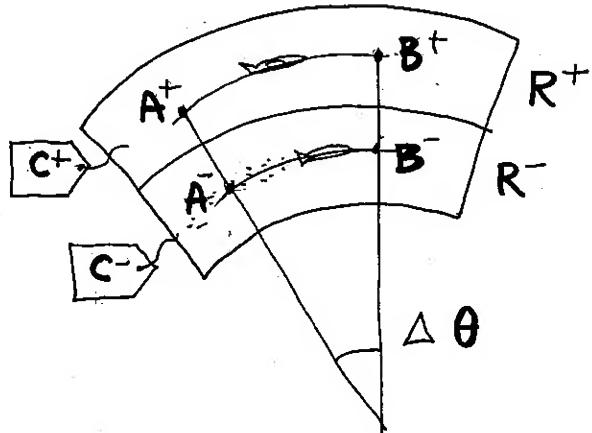
Over time, astronomical observations have begun to confirm that not everything in the Universe is optically observable and intangible - The idea thus came about, very speculative (in science lots of questions are handled by creating simple words), that there could exist hypothetical particles that "only interact very weakly with our own matter" (\*). After that one could envisage particles that only interact with our matter THROUGH THE FORCE OF GRAVITY. A spaceship made of negative masses, moving at just a few dozen km/s, could cross our planet from side to side, and even the Sun, without being incommodeated (if the speed is sufficient to avoid it becoming prisoner of the sun's gravitational field). As it is a question of travel at apparently supraluminic speeds, we can give the following image: In a "double" world supersonic displacement is impossible. But to go from one place to another, two methods are possible: by air, at less than 340 m/s and - underwater, at less than the speed of sound in this environment, which this time is ten times higher.



(\*) We called them WIMPS (weakly interacting massive particles)

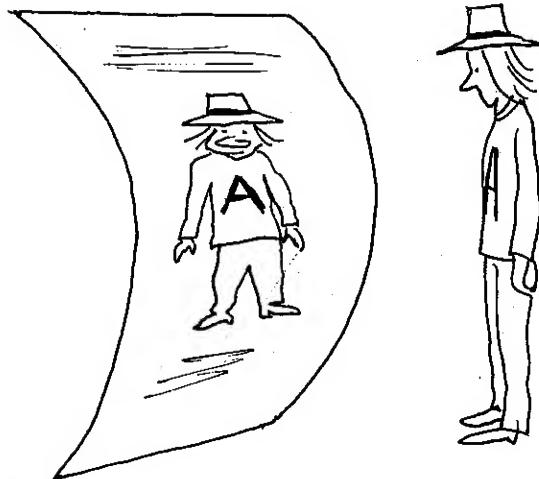
# THE GULLIVER EFFECT

As it is a question of shortening the distance to cover we can imagine that the displacement is only ANGULAR and lends itself to two different types of REPRESENTATION linked to very different space scale factors (WARP FACTORS),  $R^+$  and  $R^-$ , these spatial representations being themselves linked to the very different speeds of light  $C^+$  and  $C^-$ :

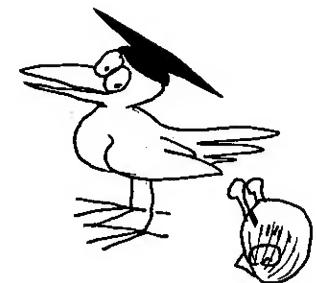


$$\left\{ \begin{array}{l} R^+ \gg R^- \\ C^+ \ll C^- \end{array} \right.$$

We thus win both ways: in the "negative world" ("twin" Universe): less distance to cover, faster.



The distances to cover are also shorter behind a concave mirror. All it needs is for me to "cross to the other side of the mirror".



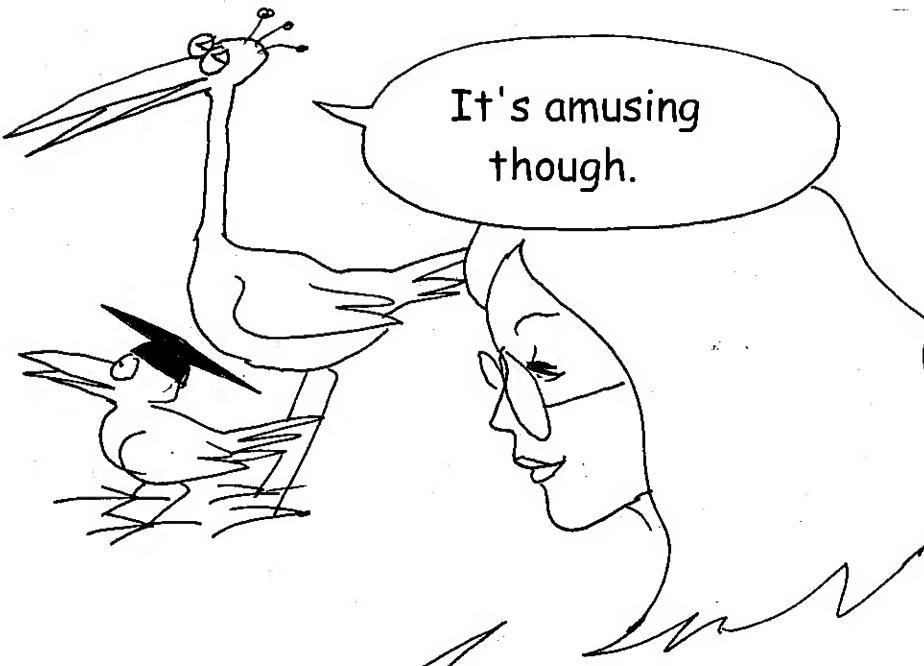
This story is getting lost, and it's starting to seem more and more like ALICE THROUGH THE LOOKING GLASS. We're wallowing in complete fiction.

But today's science is yesterday's fiction - A century earlier the direct transformation of matter into energy according to the law  $E=MC^2$  was complete fiction.

This law is nothing other than the PRINCIPLE OF CONSERVATION OF ENERGY MATTER.

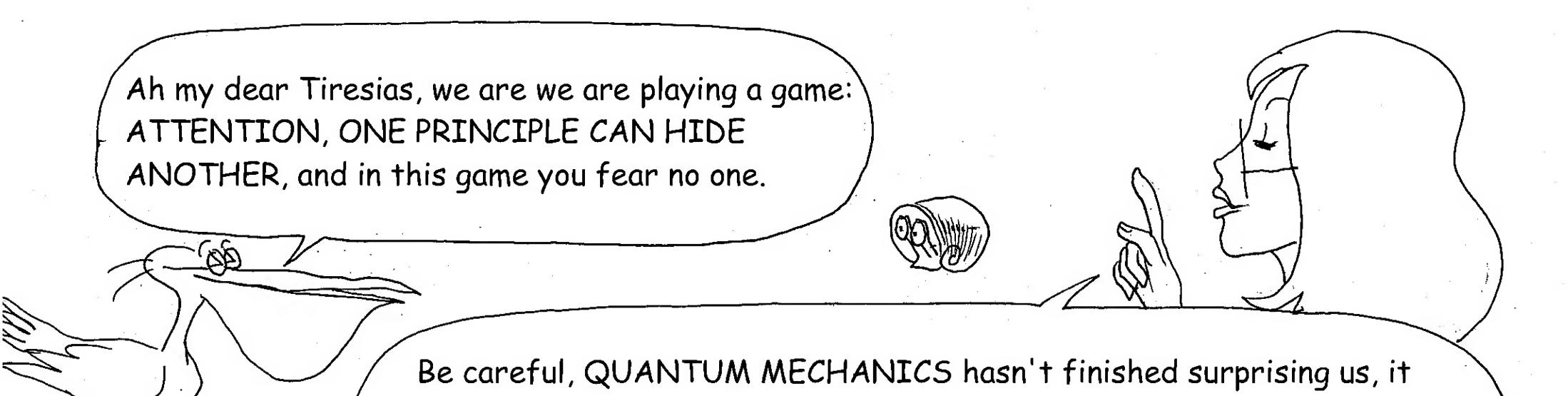
We would have said it was impossible because it violated the LAW OF CONSERVATION OF MATTER.

For this "gemellary" vision I propose a new principle: from one sheet to another  
ENERGY-MATTER IS CONSERVED.



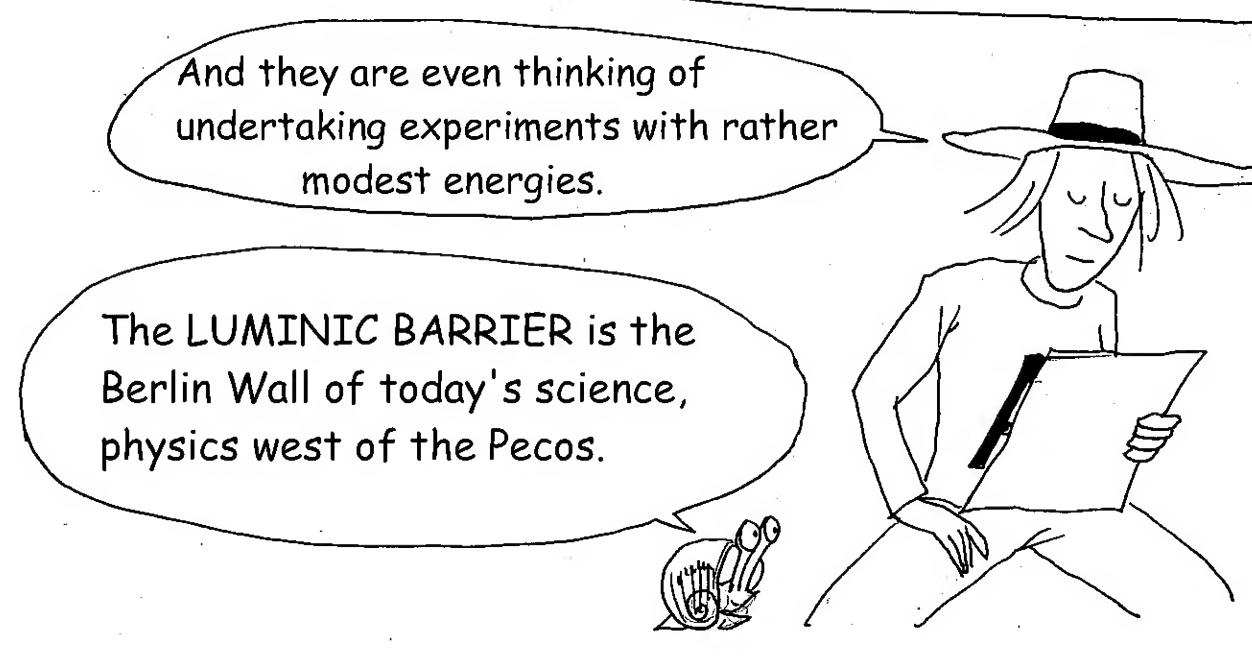
It's amusing though.





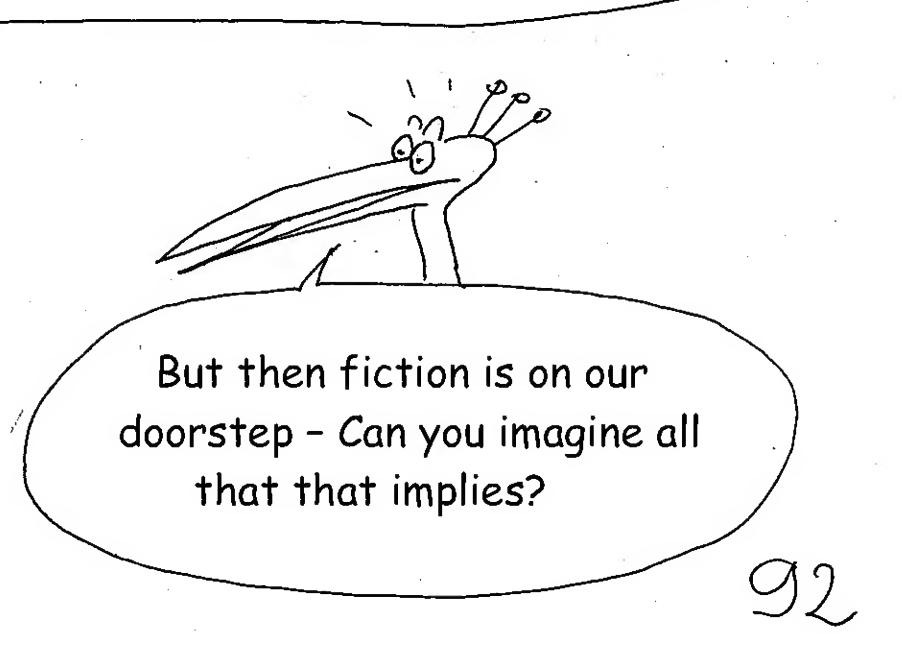
Ah my dear Tiresias, we are we are playing a game:  
ATTENTION, ONE PRINCIPLE CAN HIDE  
ANOTHER, and in this game you fear no one.

Be careful, QUANTUM MECHANICS hasn't finished surprising us, it handles the PROBABILITIES OF PRESENCE. Two researchers FABRICE PETIT and MICHAEL SARRAZIN have just published a work in the PHYSICAL REVIEW D with a TWO SHEET REPRESENTATION where a particle can pass from one sheet to another, by also bringing into play the principle of the conservation of energy-matter, the TIRESIAS PRINCIPLE.



And they are even thinking of  
undertaking experiments with rather  
modest energies.

The LUMINIC BARRIER is the  
Berlin Wall of today's science,  
physics west of the Pecos.



But then fiction is on our  
doorstep - Can you imagine all  
that that implies?

# PHYSICS WEST OF THE PECOS

## Plausible "faster-than-light" displacements in a two-sheeted spacetime

Fabrice Petit<sup>1,\*</sup> and Michaël Sarrazin<sup>2,†</sup>

<sup>1</sup> Belgian Ceramic Research Centre,

4 avenue du gouverneur Cornez, B-7000 Mons, Belgium

<sup>2</sup> Laboratoire de Physique du Solide, Facultés Universitaires Notre-Dame de la Paix,  
61 rue de Bruxelles, B-5000 Namur, Belgium

In this paper, we explore the implications of a two-point discretization of an extra-dimension in a five-dimensional quantum setup. We adopt a pragmatic attitude by considering the dynamics of spin-half particles through the simplest possible extension of the existing Dirac and Pauli equations. It is shown that the benefit of this approach is to predict new physical phenomena while maintaining the number of constitutive hypothesis at minimum. As the most striking feature of the model, we demonstrate the possibility of fermionic matter oscillations between the two four-dimensional sections and hyper-fast displacements in case of asymmetric warping (without conflicting special relativity). This result, similar to previous reported ones in brane-world theories, is completely original as it is derived by using quantum mechanics only without recourse to general relativity and bulk geodesics calculation. The model allows causal contact between normally disconnected regions. If it proves to be physically founded, its practical aspects could have deep implications for the search of extra-dimensions.

PACS numbers: 11.10.Kk, 04.62.+v, 11.25.Wx



1. arXiv:0809.2060 [ps, pdf, other]

## Probing braneworlds through artificial matter exchange between branes: experimental setups for neutron and helium-3 disappearance

Michael Sarrazin, Fabrice Petit, submitted

2. arXiv:0706.4025 [ps, pdf, other]

## Plausible "faster-than-light" displacements in a two-sheeted spacetime

Fabrice Petit, Michael Sarrazin. Accepted for publication in Phys. Rev. D 76, (2007)

Journal-ref: Phys. Rev. D 76, 085005 (2007)

3. arXiv:hep-th/0603194 [ps, pdf, other]

## Matter localization and resonant deconfinement in a two-sheeted spacetime

Michael Sarrazin, Fabrice Petit. Accepted for publication in Int. J. of Modern Physics A 22 (2007) 2629-2641

4. arXiv:hep-th/0505014 [ps, pdf, other]

## Artificially induced positronium oscillations in a two-sheeted spacetime: consequences on the observed decay processes

Michael Sarrazin, Fabrice Petit. Accepted for publication in Int. J. of Modern Physics A 21 (2006) 6303-6314

5. arXiv:hep-th/0409084 [ps, pdf, other]

## Quantum dynamics of massive particles in a non-commutative two-sheeted space-time

Fabrice Petit, Michael Sarrazin. Accepted for publication in Physics Letters B 612

6. arXiv:hep-th/0409083 [ps, pdf, other]

## Quantum dynamics of particles in a discrete two-branes world model: Can matter particles exchange occur between branes?

Michael Sarrazin, Fabrice Petit. Published in Acta Physica Polonica B (2005)

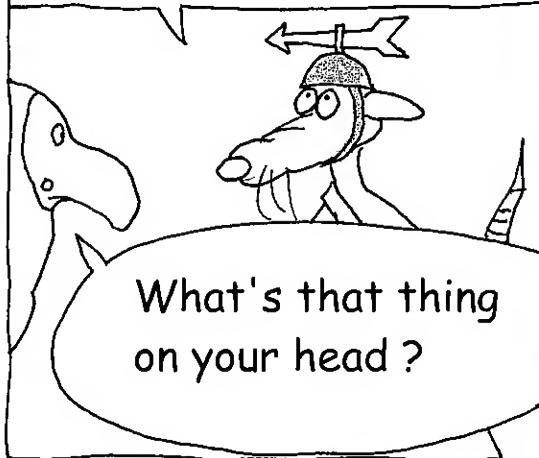
Journal-ref: Acta Phys. Polon. B36 (2005) 1933-1950

Kiss, what do you think of these tales of particles which jump from one sheet to another ?



It all depends on the consensus my dear Handshic. If a large consensus appears, Main Stream will follow the movement.

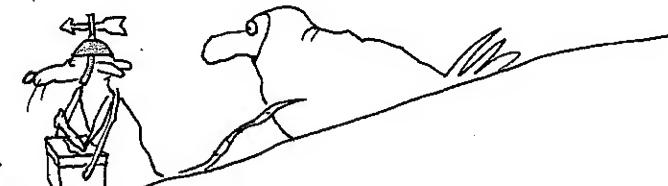
When the weather changes, we change with it.



It allows us to know in which direction the wind of science is blowing.



An opinion! And what next? As if life wasn't already complicated enough!



Ah sorry. I can just see Professor Nostradamour over there coming out of the Institute. He's an uncontested specialist in superstrings. I'm off to interview him. Do you understand, I'm in favour of anything that helps sales.



Good old Harvey Kiss, with his review  
Main Stream under his arm, his shoe  
shine equipment and now the hat, he'll  
make me die laughing !

OK, let's recapitulate. We live in a double world filled with particles of opposed masses and energies. The Group Master says: it's normal. It's because they travel backwards in time. And in addition, the distances covered between one point of this universe and another, differ according to whether it is constituted of positive or negative masses.

I admit, I'm lost !



Ahem...

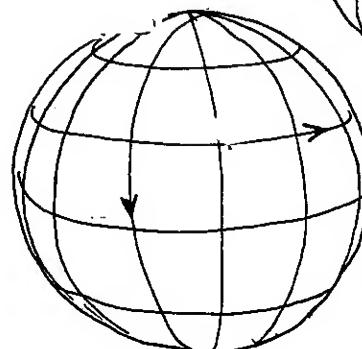
How do we make the regions with opposed time arrows interact and where the methods for measuring lengths are also different ?!?

# SPACE-TIME TOPOLOGY

You just need to fold the Universe on itself.

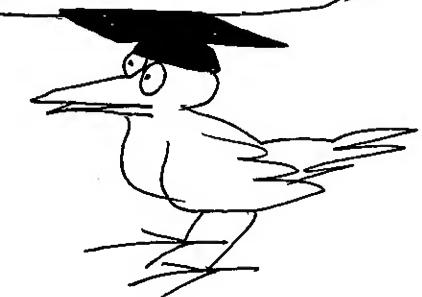
Start with a space-time model with a BIG BANG, a BIG CRUNCH and a situation of maximum extension, that you could represent in 2D with the help of a simple sphere.

What is this madman's story?



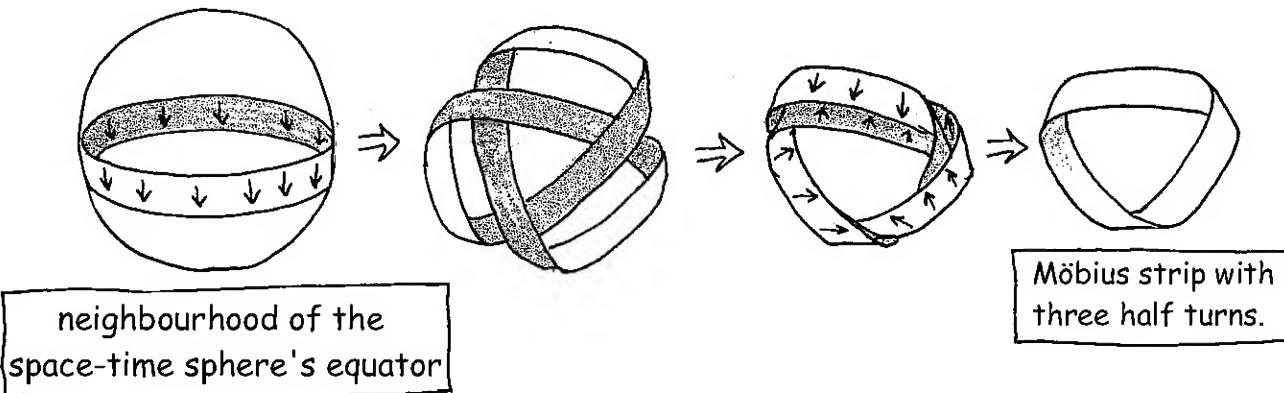
You know that you can bring every point of this space-time into coincidence with its SPATIO-TEMPORAL ANTIPODE (the antipodal point on the sphere  $S^2$ ) and the result is a Boy surface. That's all explained in the album TOPO THE WORLD (\*).

Yes, time follows the meridians and space, which has just one dimension, is shown by a parallel circle which starts at zero, at the "BIG BANG pole", grows until it becomes the sphere's equator then collapses according to the "BIG CRUNCH pole".

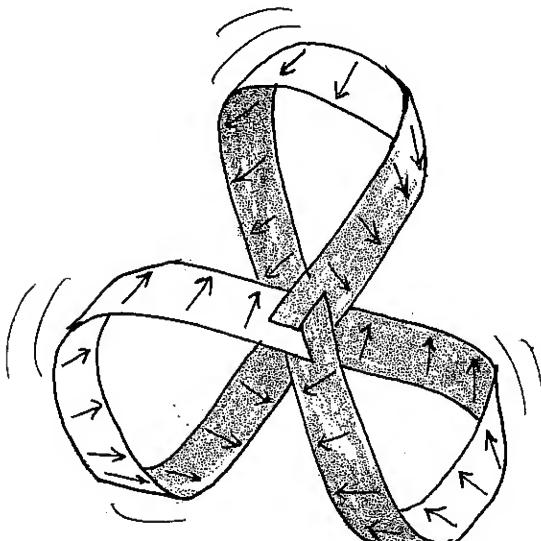
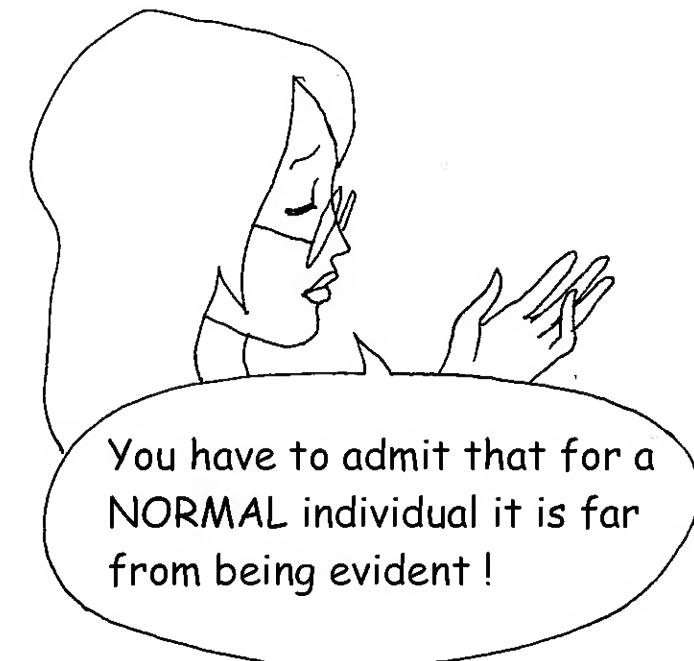


(\*) Have a look there, we can't go over it all again here.

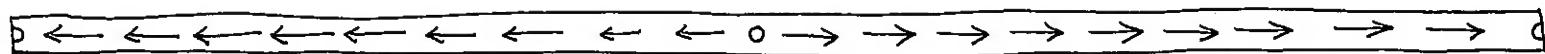
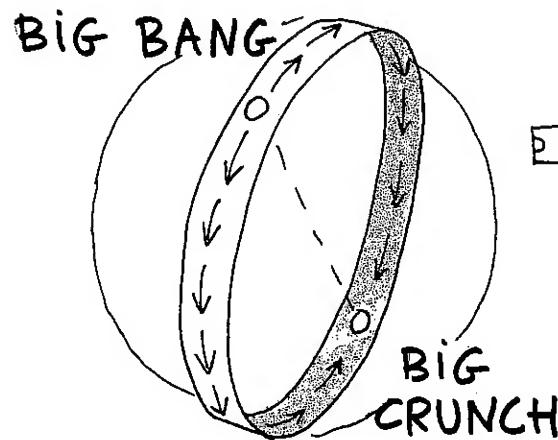
In flicking backwards through pages 71 to 43 four animations allow us to follow the folding near the equator which, by bringing the antipodal points into coincidence, shows how the regions with opposed time arrows form a TWO SHEET SPACE-TIME.



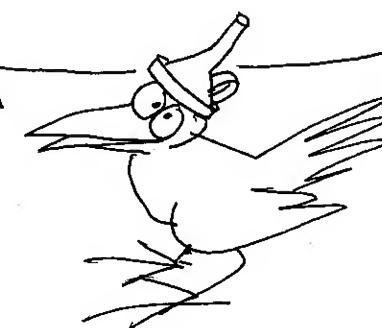
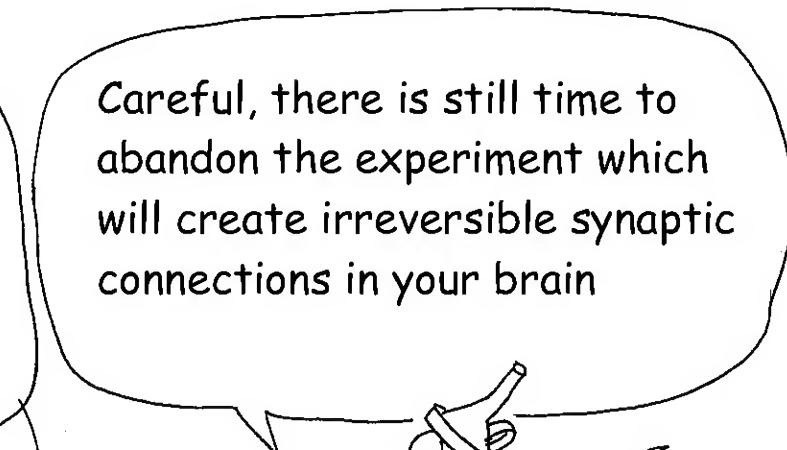
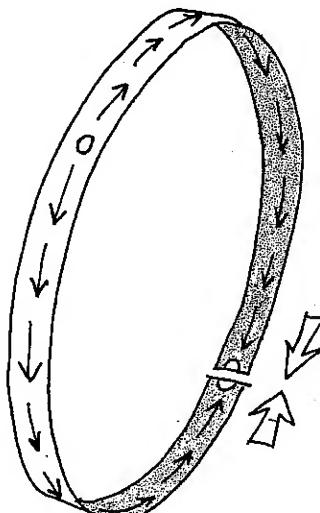
The area near the equator is configured according to the COATING of a Möbius strip with three half turns. But this operation is hard to do yourself, three layers need to be crossed, as shown in the figure on page 59:



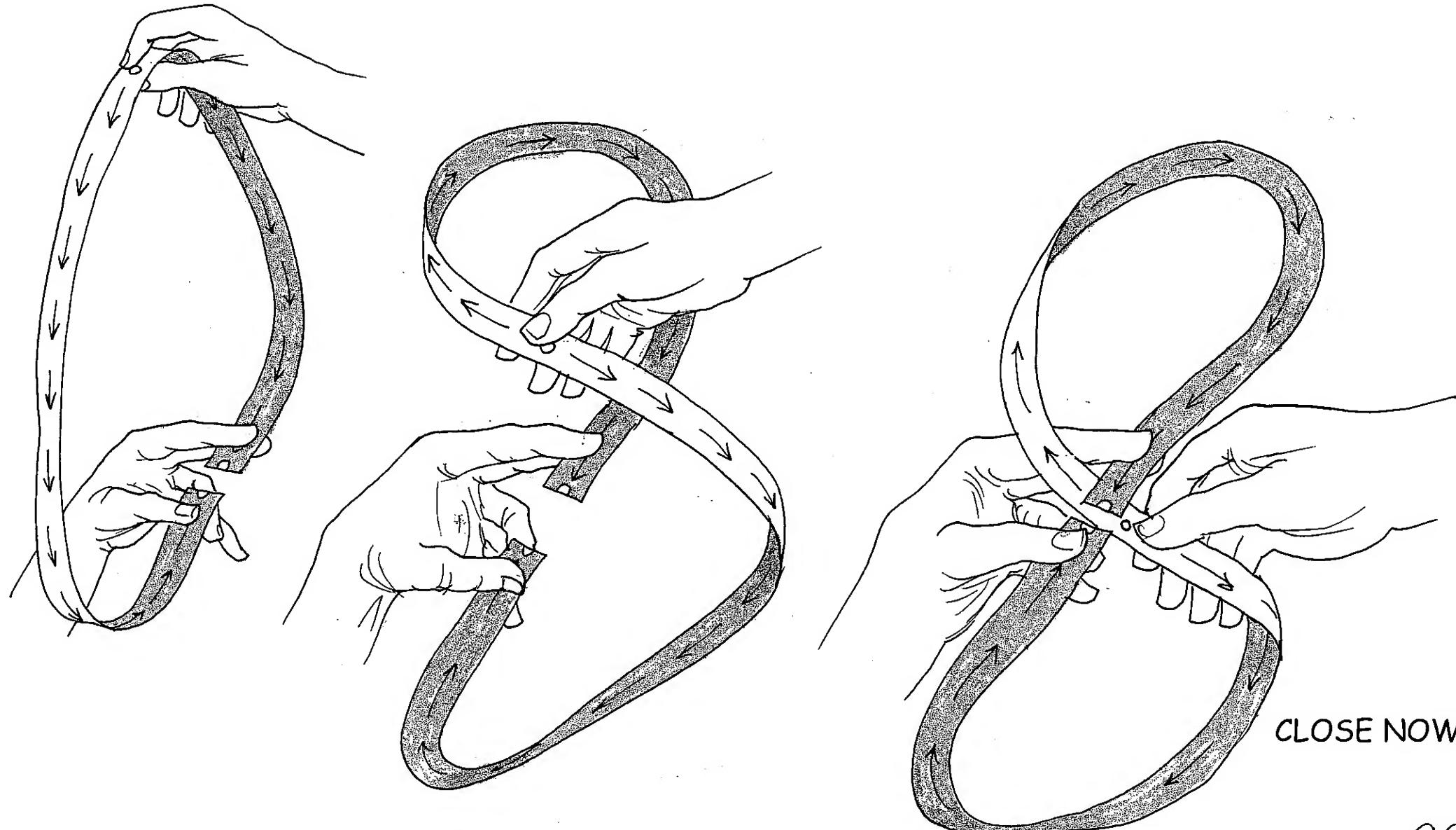
To show you how the space-time sphere folds onto itself, bringing the points into coincidence with their antipodes and at the same time bringing two regions with opposed time arrows "face to face", we'll proceed differently. This time we'll start near a MERIDIAN of our two dimensional space-time sphere - Make a long strip of paper, 2cm wide and just over 80cm long - In the middle draw a circle which represents the BIG BANG and from one end to the other the time arrows. At the end of the strip draw two small half-circles.



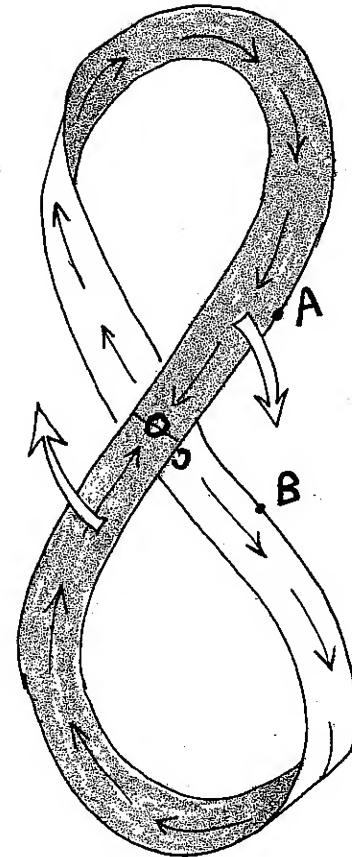
Make identical marks on the other side of the strip. It's ready when the two extremities are joined, to be done near a time line, which we'll call UNIVERSE LINE.



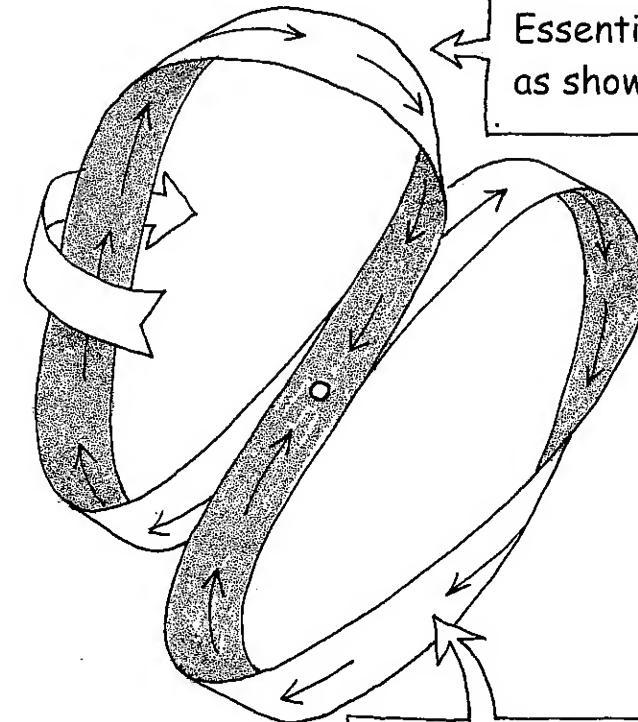
The folding of the sphere according to the covering of two sheets of a Boy surface (\*) can't be done without the surface cutting itself. Therefore we will operate A crossing before closing the BILATERAL band with adhesive tape then continuing as follows:



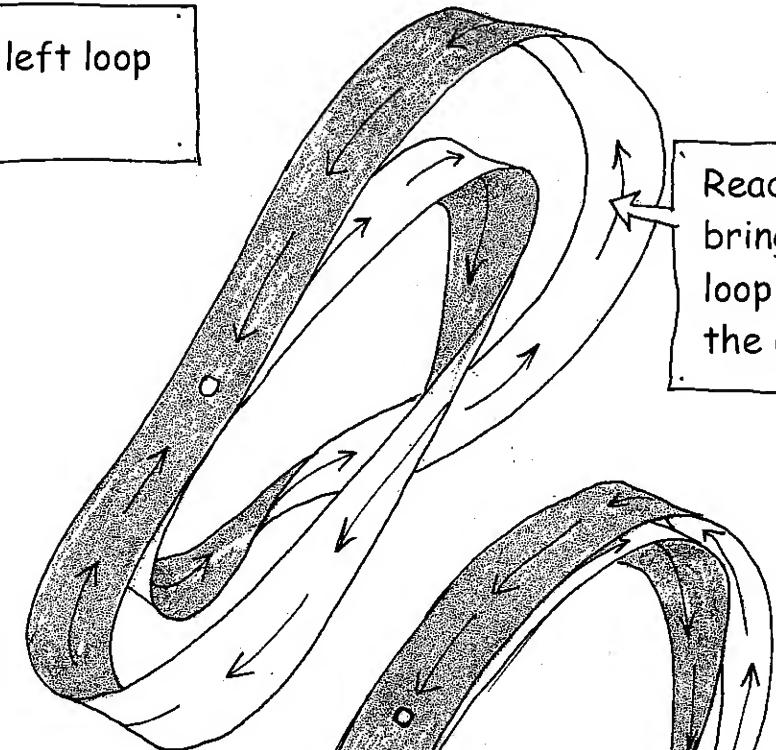
(\*) Look at "TOPO THE WORLD"



Hold that the two "poles", the BIG BANG and the BIG CRUNCH one on the other between your thumb and index finger. Then you effect a "scissor" rotation bringing point A onto point B.



Essential: turn the left loop as shown.



Ready to bring one loop onto the other.

You can see that this loop is a Möbius strip with half a turn.

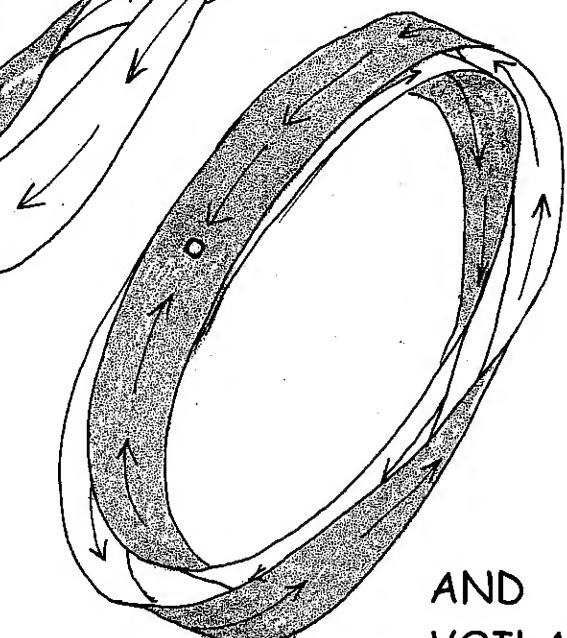


This band is BILATERAL.

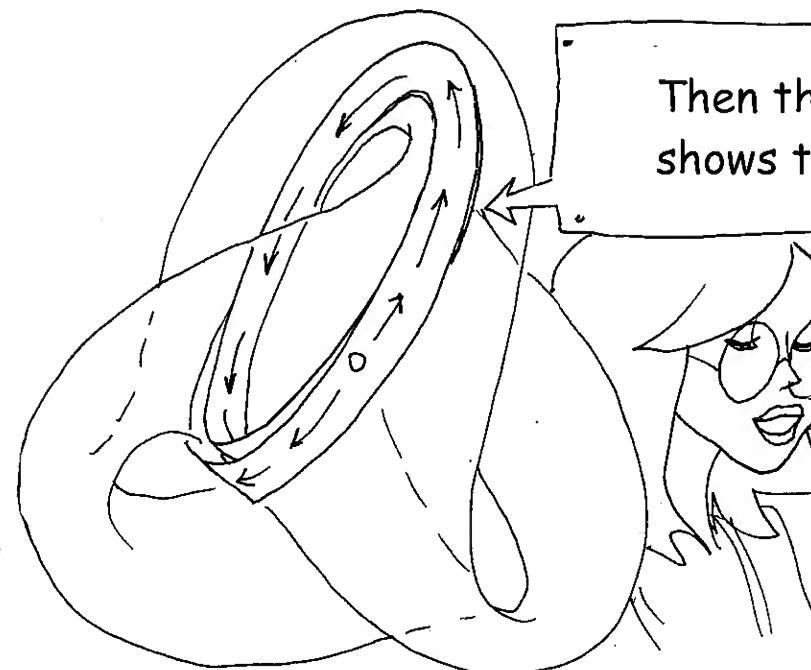
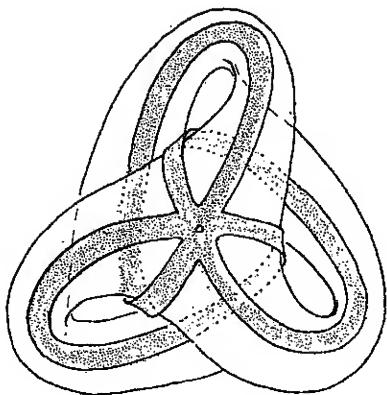
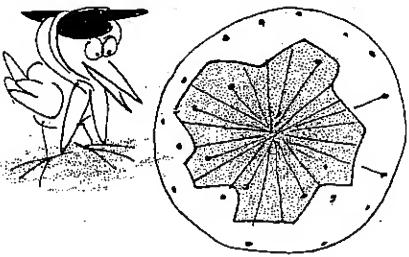
To keep that clear in your head pencil one of the two faces in grey.

You'll see that this operation puts the white side against...itself.

This done, find the gesture that will, in a trice, make the grey side disappear this time, without cutting anything!



AND  
VOILA !



We largely developed the theme of coinciding the points of a sphere with their antipodal counterparts in TOPO THE WORLD thirteen years ago. There the meridians of the sphere, the UNIVERSE LINES of a spherical space-time  $S^2$ , "folded" according to the two sheet covering of a Möbius strip with three half-turns. Here are three of these folded meridians:

Then the object that we've just created shows the inversion of the time arrow.

What we do with an  $S^2$  sphere can also be done with an  $S^4$  sphere (\*)

NOTE : If, instead of putting the "pole BIG BANG" up against the "pole BIG CRUNCH", we envisaged a tubular passage, by eliminating the SINGULARITY our space-time now becomes toric and "folds" according to the two sheet covering of a Klein bottle, with an unusual aspect.

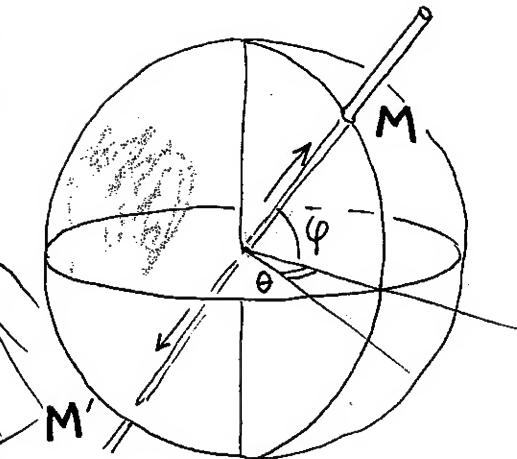
In other words, this interplay between positive and negative masses is a consequence of the topological configuration of the Universe.



(\*) A hyperspherical space-time, firm, "compact".

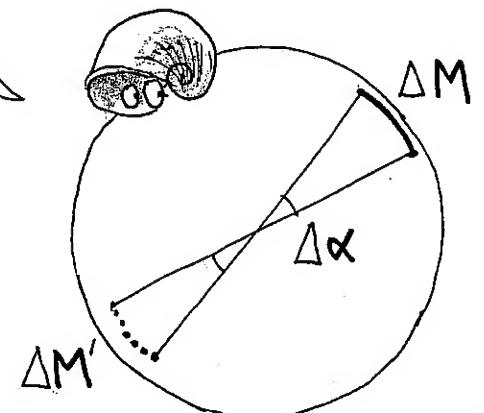


Ok, I can conceive that this inversion of time (so of mass) is another case of a geometric practical joke, one more. But what about DISTANCES?



When you associated the antipodal regions of your sphere it's as if you used fibre optics and made them emit light at both ends. Each fibre is situated by ANGULAR coordinates  $(\theta, \varphi)$ . It doesn't design one point of the sphere but two, antipodal,  $M$  and  $M'$ .

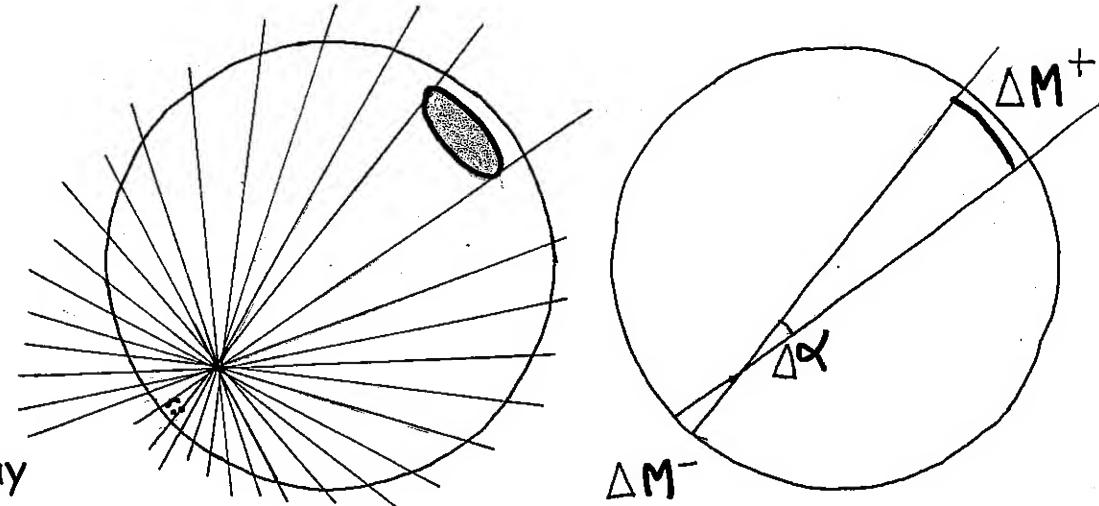
displacement corresponding to an ANGULAR VARIATION  $\Delta\alpha$  with which are associated two TRAJECTORIES  $\Delta M$  and  $\Delta M'$ , which being  $\Delta M = R\Delta\alpha = \Delta M'$  will be equal if the screen projection system is at the centre of the sphere.



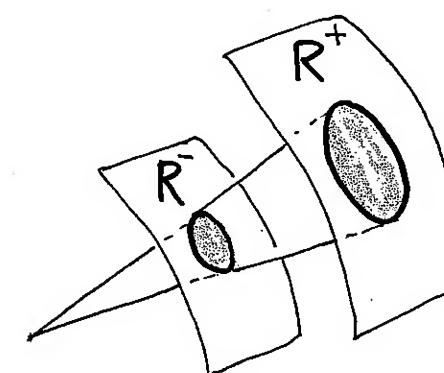
If the "projection system" is "off centre" then the same displacement  $\Delta\alpha$  (a "place" defined by angles) will not correspond to the same distance covered if it is inscribed on the "negative mass screen". The phenomenon perceived as expansion is, in fact, the variation of the warp factor  $R$  according to time. This isn't "lived", that is to say MEASURED in the same way by the two sub-groups.

The system is UNSTABLE. If the warp factor  $R^+$  of the positive masses increases more quickly than the warp factor  $R^-$  of the negative masses then this movement accelerates. However, beings who lived in this NEGAWORLD would be subjected to a deceleration (curves). It is this phenomenon that is wrongly attributed to the REPULSIVE POWER OF THE VOID or to DARK ENERGY

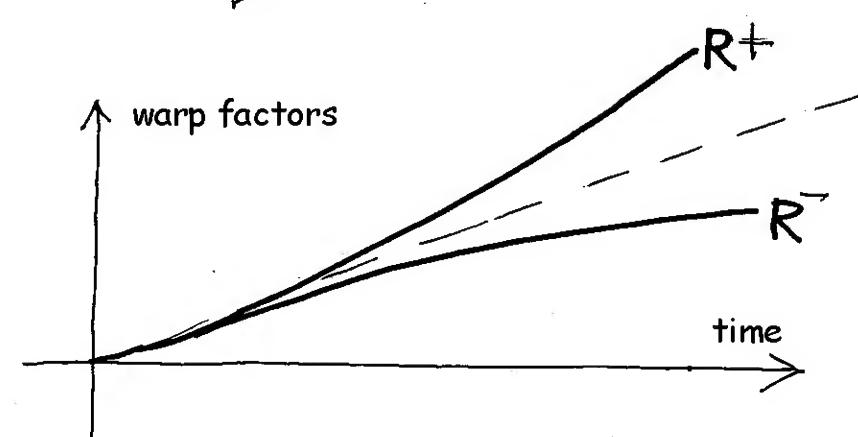
(see annex)



$$\Delta M^+ = R^+ \Delta \alpha > \Delta M^- = R^- \Delta \alpha$$



$$\left. \begin{aligned} \rho^+ &\sim \frac{1}{R^{+3}} \\ \rho^- &\sim \frac{1}{R^{-3}} \end{aligned} \right\} \text{(density)}$$

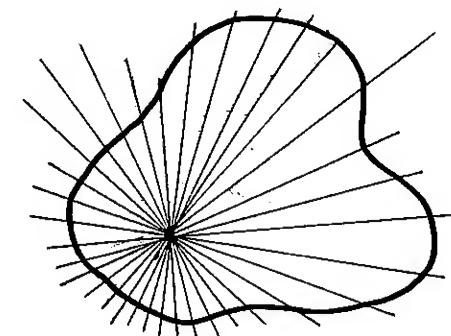


# THE FAILURE OF COSMOLOGICAL HYPOTHESES

The STANDARD COSMOLOGICAL MODEL is based on a certain number of FUNDAMENTAL HYPOTHESES that no one would dream of calling into question :

- THE UNIVERSE IS A CONTINUUM (which more and more people are questioning).
- THE UNIVERSE IS HOMOGENOUS (false: its structure is LACUNAR) (\*)
- THE UNIVERSE IS ISOTROPIC (increasingly contradicted by observation).
- THE CONSTANTS OF PHYSICS ARE ABSOLUTE CONSTANTS (\*)

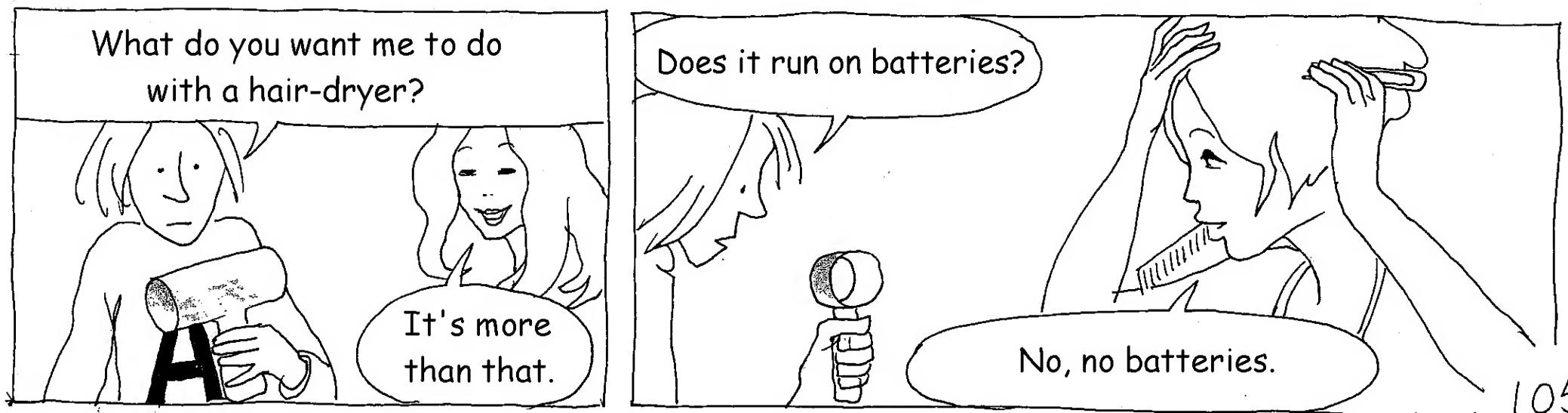
The shadow of things cannot be projected into the cave on just one wall but two: the shadows interact. The projection system isn't at the centre and, to finish, it is likely that these "walls" oscillate, warp, the phenomenon showing itself through ANISOTROPIES.



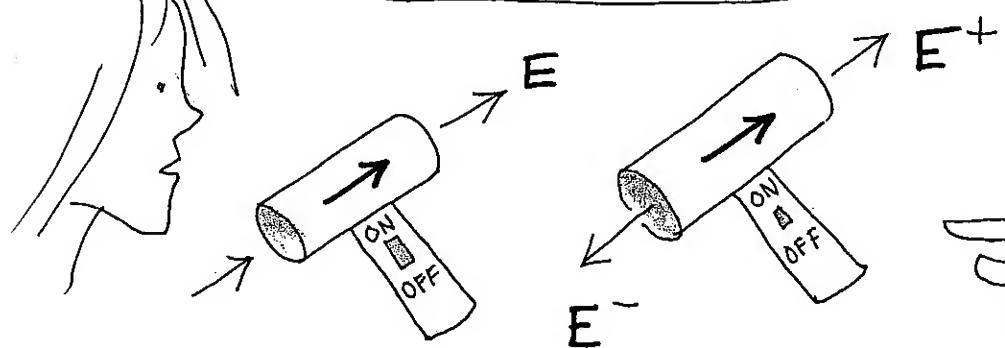
In short,  
it's all screwed up.

(\*) On this matter see FASTER THAN LIGHT

# CONJOINED GEOMETRIES (\*)

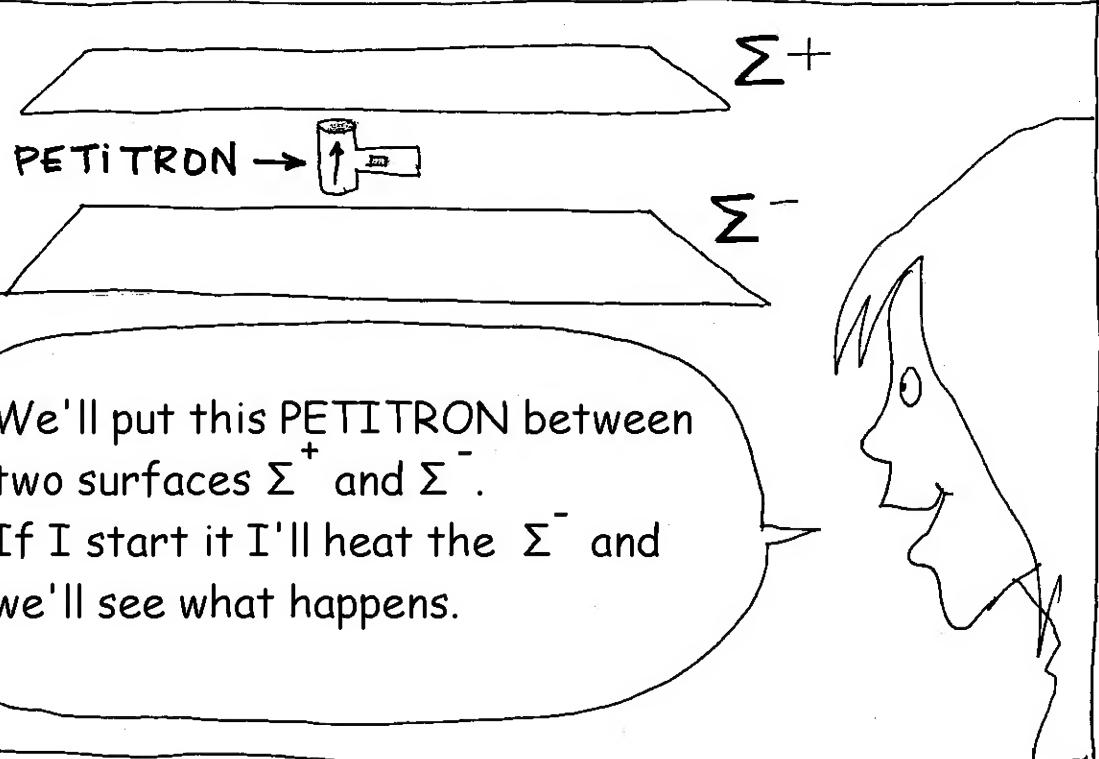
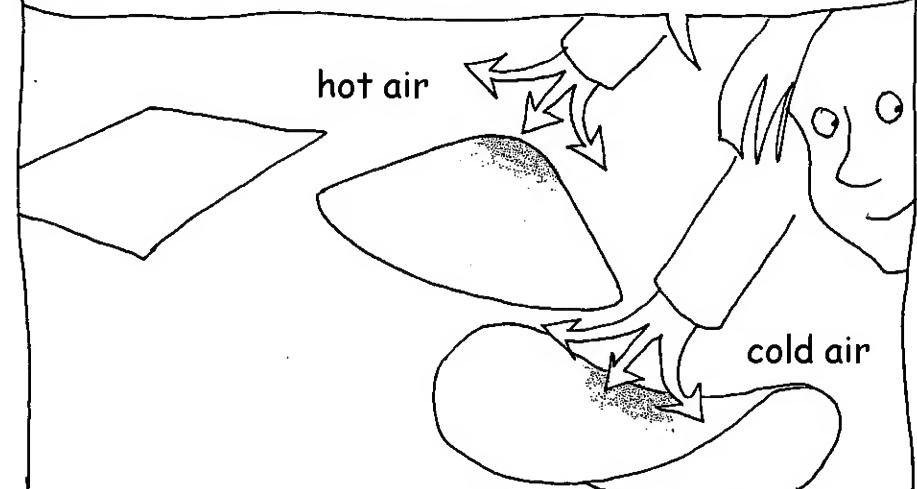


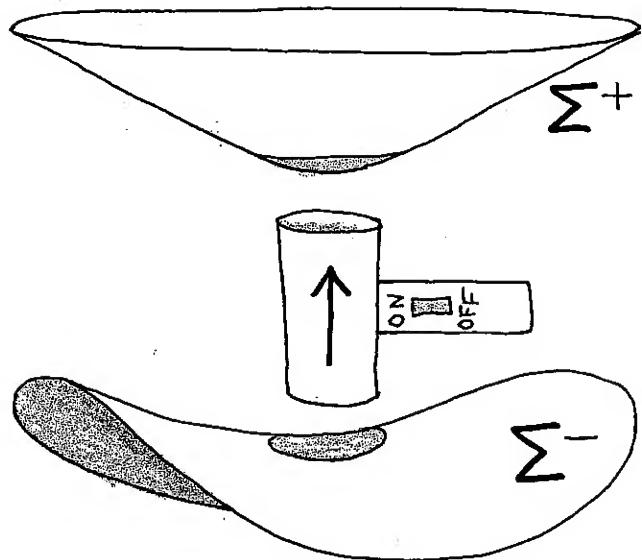
It was invented in 1994 by a Frenchman,  
it's called a PETITRON(\*)



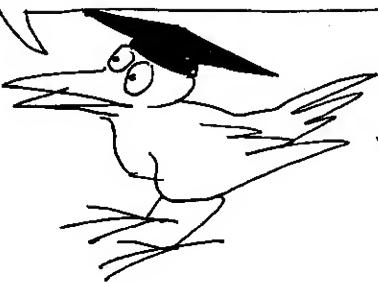
It takes energy from one side and sends it out of the other, in equal quantity - in that way I can dry my hair and chill your soup at the same time.

Wait, that gives me an idea. Do you remember that when blowing either cold air or hot air onto a metal sheet we created NEGATIVE or POSITIVE curves?

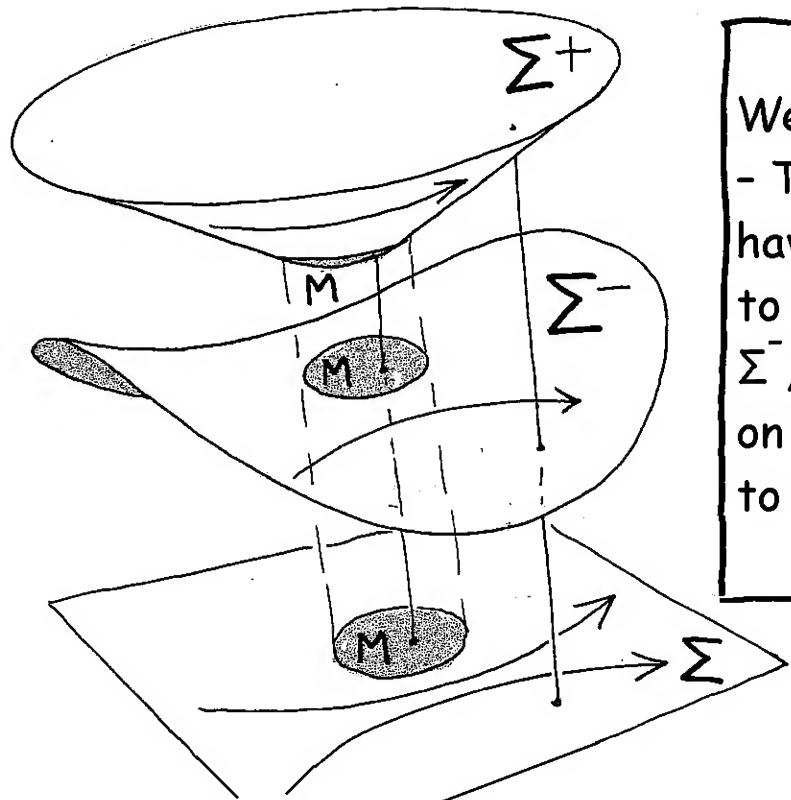




Simple, you create a BLUNTED POSICONE on the surface that receives POSITIVE ENERGY and a BLUNTED NEGACONE on the surface that receives NEGATIVE ENERGY. And as CURVATURE equals ENERGY we will have two regions, face to face, containing equal CURVATURE QUANTITIES but with OPPOSED SIGNS.



We call that  
CONJOINED GEOMETRIES.

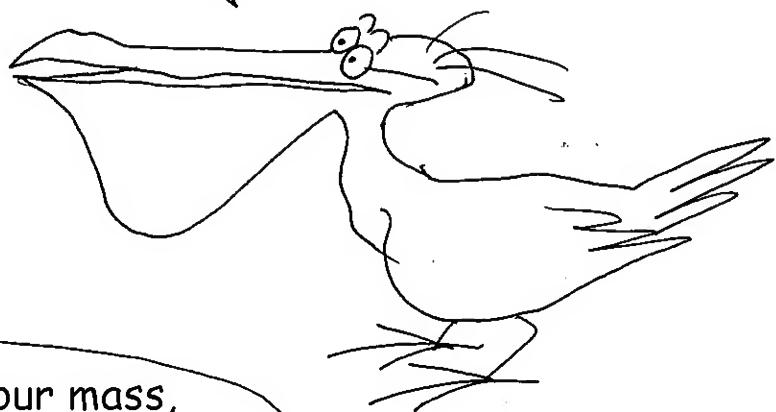


We can associate the points  $M^+$  and  $M^-$  of the two surfaces  
 - The grey regions have opposed curvatures. The white regions have nil curvatures - That is two points  $M_1^+$  and  $M_2^+$  belonging to  $\Sigma^+$  and  $(M_1^-, M_2^-)$  their COMBINED POINTS, on the surface  $\Sigma^-$ , the GEODESIC ARCS  $M_1^+ M_2^+$  and  $M_1^- M_2^-$  are not projected on to the plane  $\Sigma$ , a EUCLIDEAN representation according to the SAME CURVES.

The two surfaces  $\Sigma^+$  and  $\Sigma^-$  are the two "caves" of Plato<sup>2</sup>. The plane  $\Sigma$  is the EUCLIDEAN REPRESENTATION that we make of the world - Observers made of opposing masses SEE things totally differently - What is PRESENCE for one is ABSENCE for the other (\*).

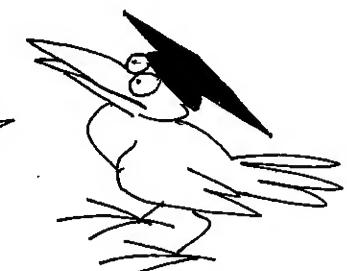


So what is REAL ?

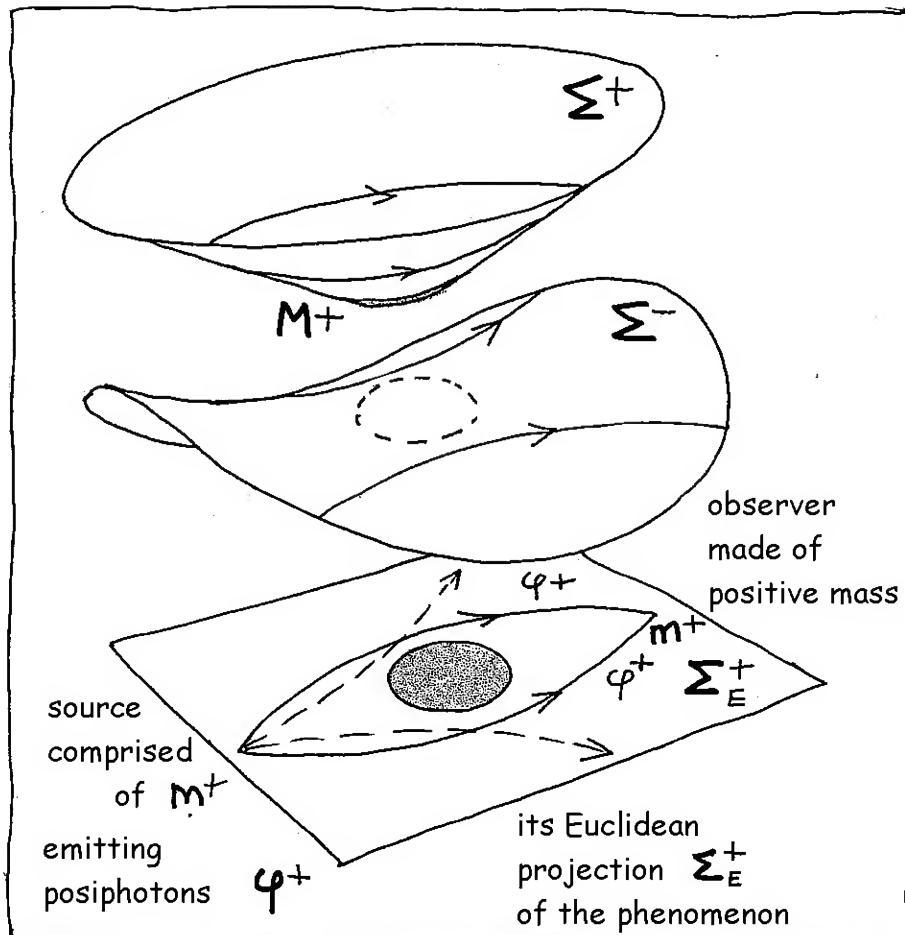


It depends on the sign of your mass,  
the way in which you project the phenomena  
in YOUR world.

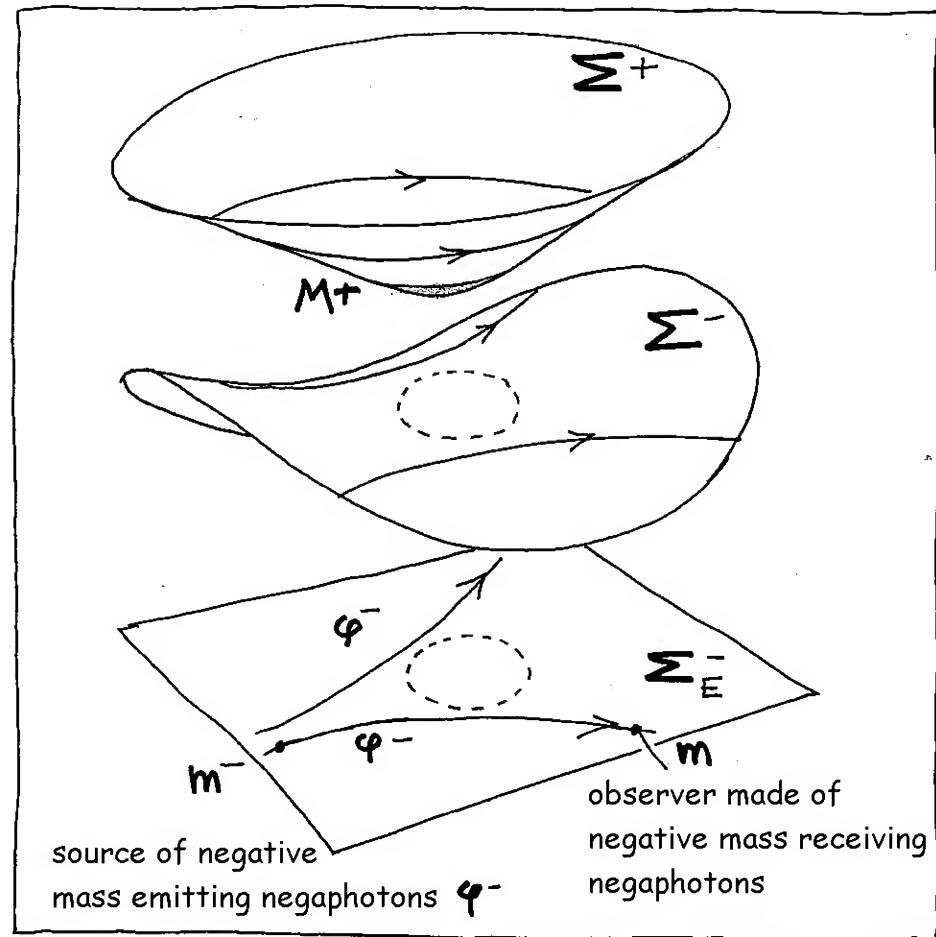
Let's go back to the previous figure. Suppose that you are made of positive mass. You will only perceive the projections of the sheet  $\Sigma^+$  on your Euclidian representation  $\Sigma$ . You will only perceive positive energy photons which follow  $\Sigma^+$  geodesics in this BIMETRIC MODEL  $(\Sigma^+, \Sigma^-)$



(\*) from the QUANTUM point of view is a POSSIBILITY OF PRESENCE for an observer  
\* made of positive mass will become a PROBABILITY OF ABSENCE in our NEGAWORLD.



An observer made of positive mass  $m^+$  will observe a **POSITIVE GRAVITATIONAL LENSING EFFECT** which affects the **POSIPHOTONS**, which alone that can make their retina and measuring instruments react.

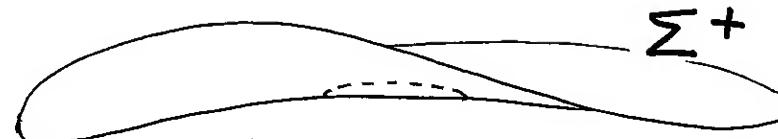
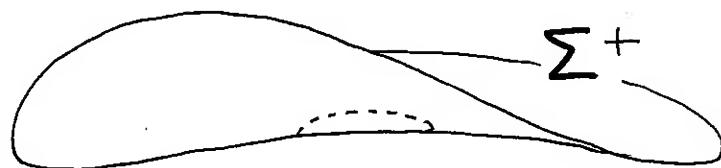


An observer made of negative mass  $m^-$  will observe a **NEGATIVE GRAVITATIONAL LENSING EFFECT** affecting **NEGAPHOTONS** which alone that can make their retina and measuring instruments react.

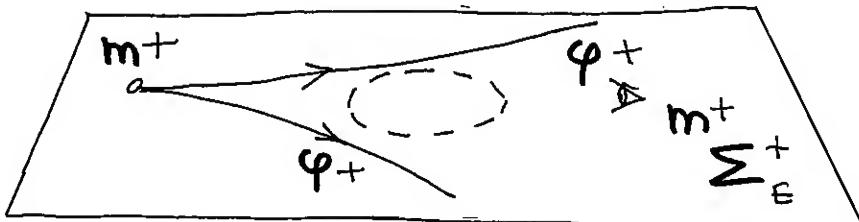
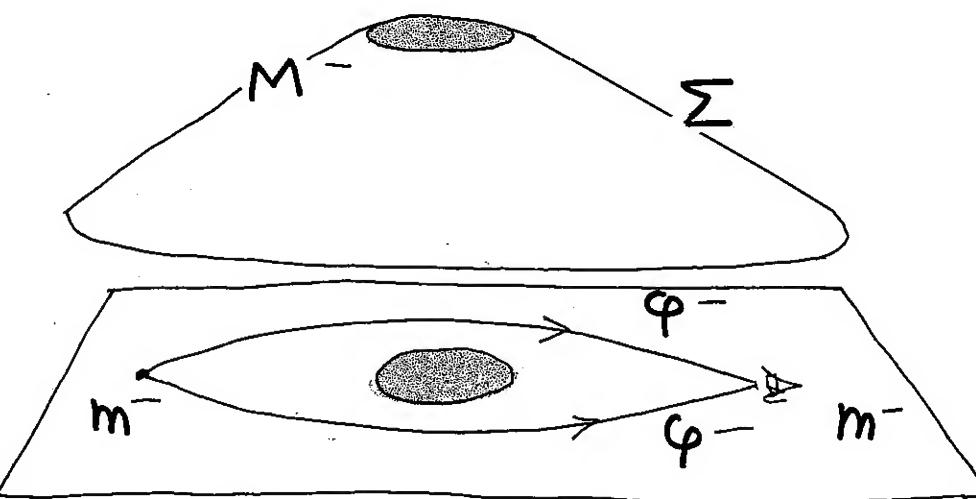
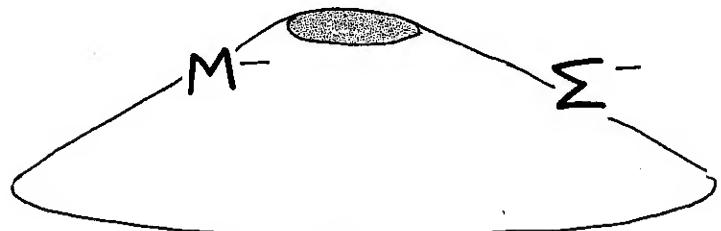
And what if we are dealing with a negative mass  $M^-$  ?

Simple = you just need to invert the figures.

# CONCEPT OF APPARENT MASS

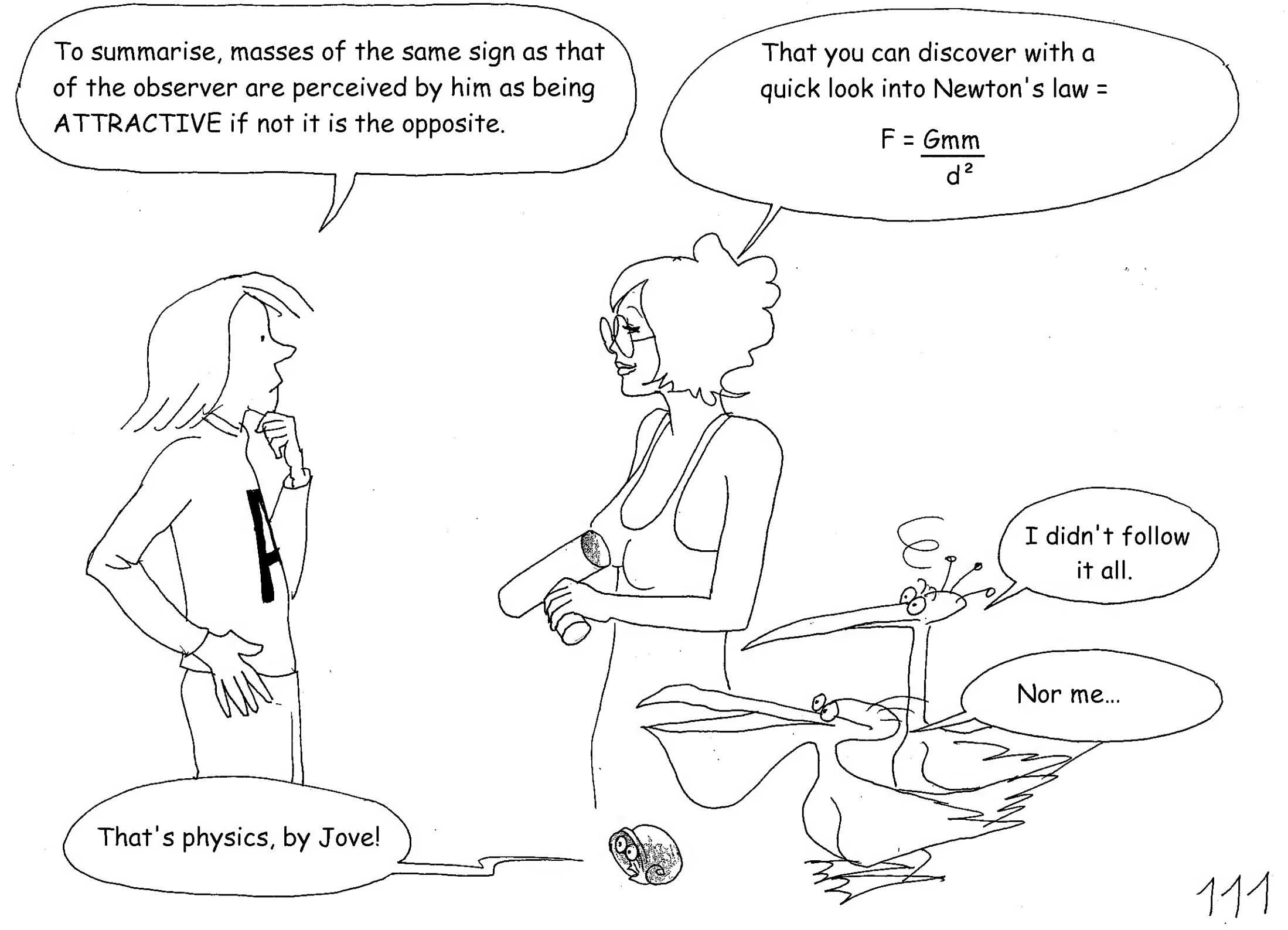


negative mass



observer made of positive mass :  
negative gravitational lensing effect

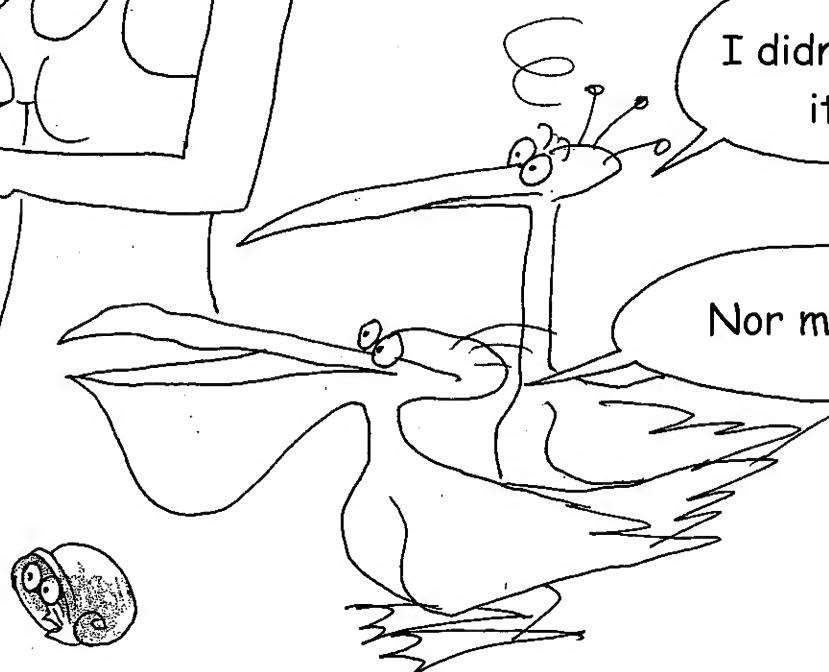
observer made of negative mass :  
positive gravitational lensing effect



To summarise, masses of the same sign as that of the observer are perceived by him as being ATTRACTIVE if not it is the opposite.

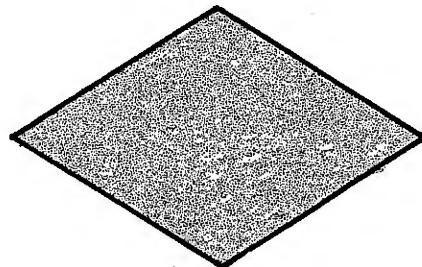
That you can discover with a quick look into Newton's law =

$$F = \frac{Gmm}{d^2}$$



# EPILOGUE

To finish we are going to propose a little exercise to illustrate that what is a positive curvature for one, is a negative curvature for another - For this we are going to imagine a world filled with people of positive and negative masses forming a regular paving. You just need to assemble the lozenges in cardboard to form an alternation of POSICORNERS and NEGACORNERS.

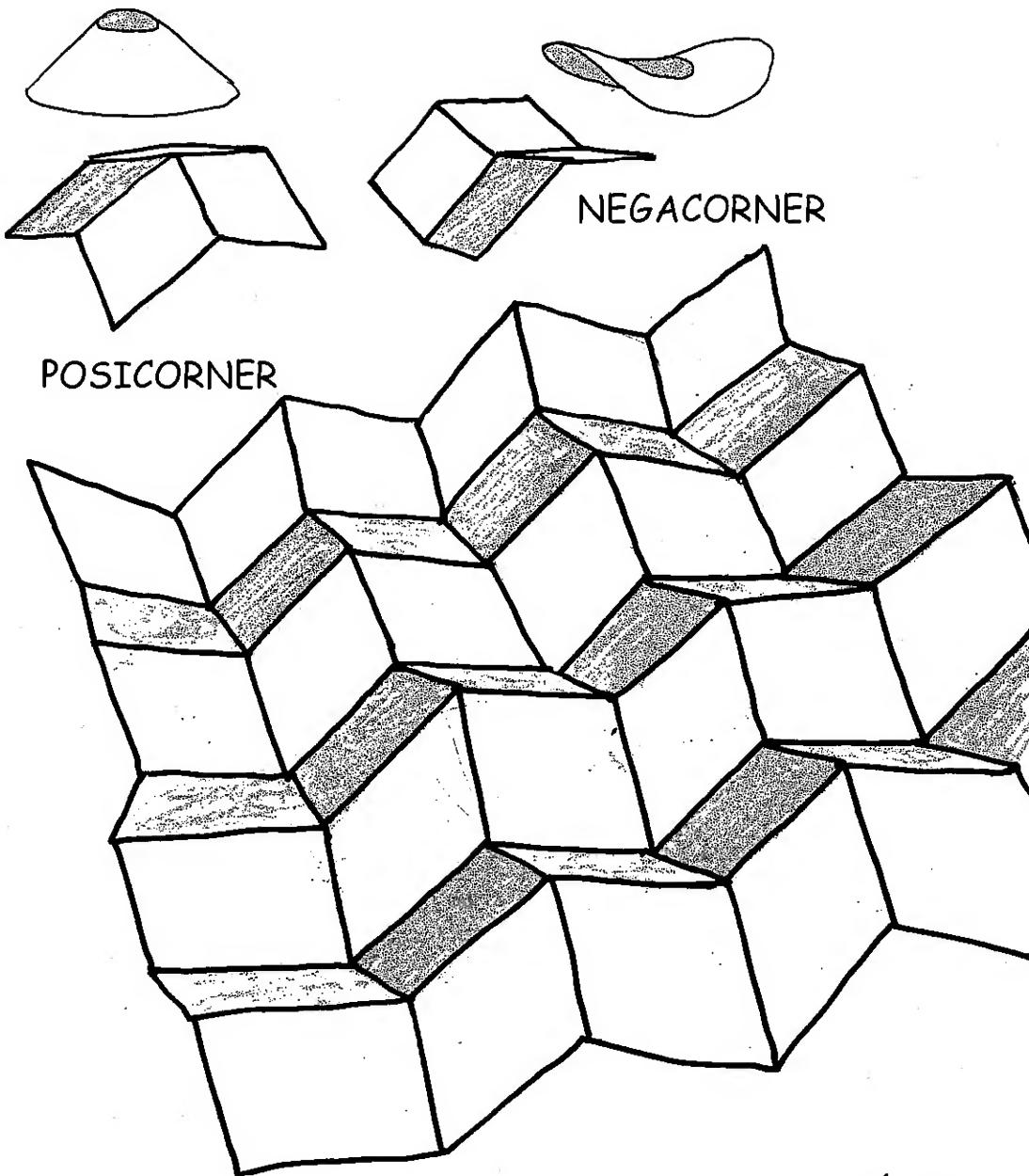


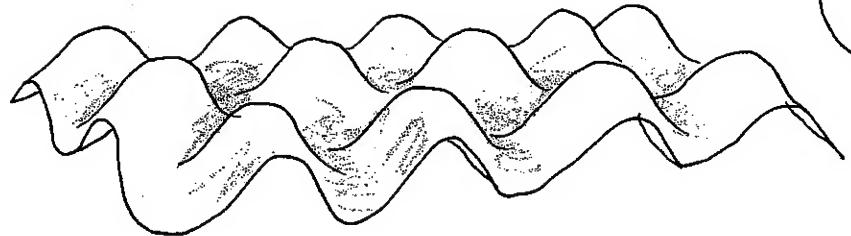
You will be constructing the POLYHEDRIC REPRESENTATION opposite.

*The Management*

POSICONE

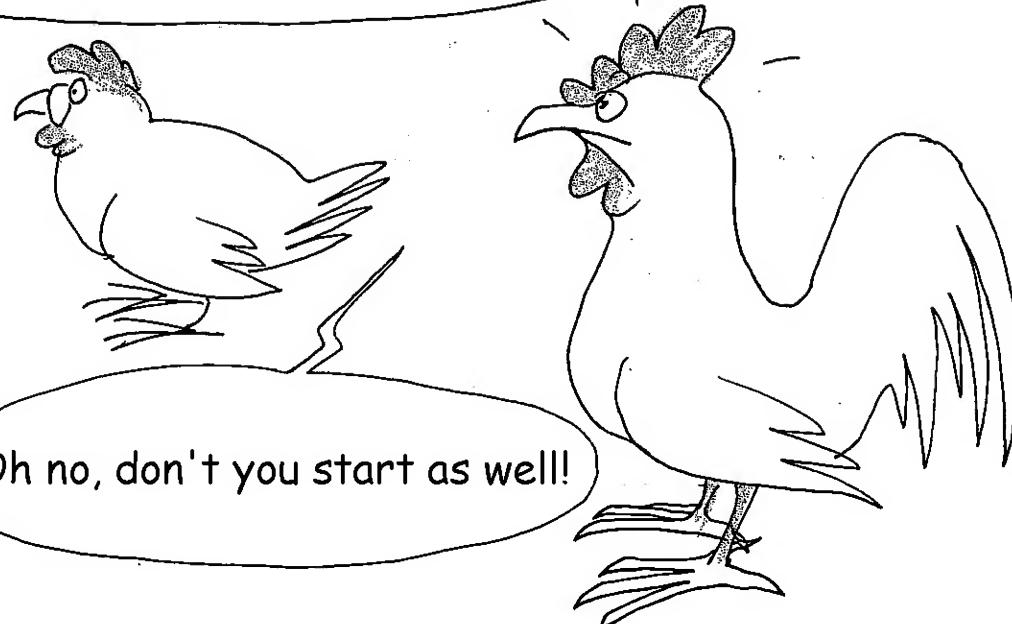
NEGACONE





By moving two of these structures you will be putting the posicorners and negacorners face to face.

It's like racks designed for putting eggs laid by POSICHICKENS and NEGACHICKENS.



Oh no, don't you start as well!

There are plenty of other things to tell you, for example on DISCRETIZING the caves (of PLATO)<sup>2</sup> but, as Kipling said...



That's another story

**THE END**

# Appendix 1

# GOD'S POLYHEDRON

Science in our times is extremely mediatized. As soon as we evoke an idea, a project, we quickly give it a touting name which will grasp people's imagination. Fifty years ago, the object which we imagined could describe the destiny of a neutron star which mass, because of the influx caused by the stellar wind originating from a companion star, could exceed the critical value of 2.5 solar masses, was called "SCHWARZSCHILD'S BODY" (\*). Not a very selling name. The word "COLLAPSAR" didn't have much success either. But when John Archibald Wheeler proposed "BLACK HOLE", its success was immediate and worldwide. Same thing for TOE (Theory of Everything), the "M THEORY" from the superstrings people. Currently our modern plutophysicists (from ploutos which signifies "wealth" in Greek) are searching for the Higg's boson, already nicknamed "GOD'S PARTICLE".

To go along this imbecile fashion and make you smile, here's the polyhedron which has only one face and one edge. We remind you that "hedron" in Greek means "face".

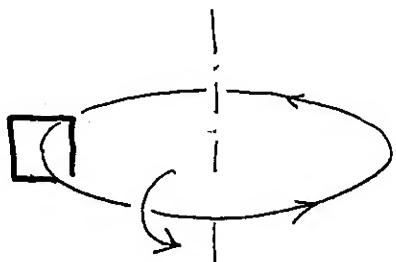
So here's the MONOHEDRON or ... "GOD'S POLYHEDRON".

*The management*

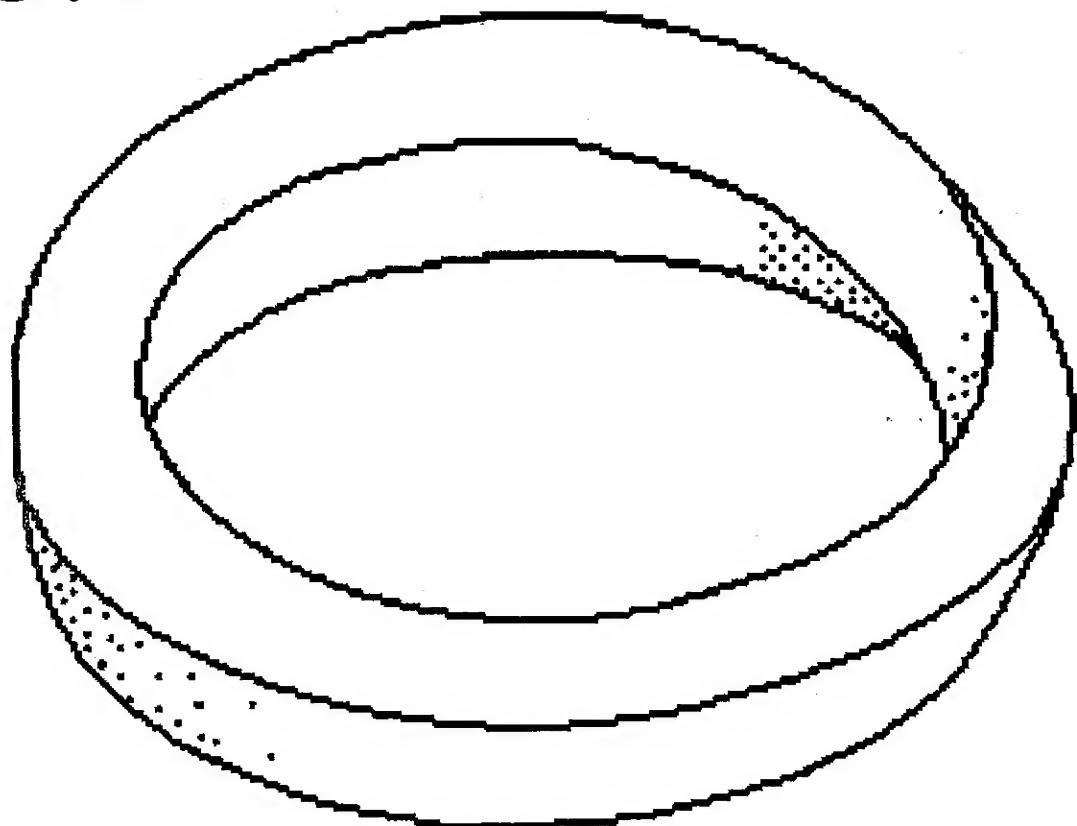
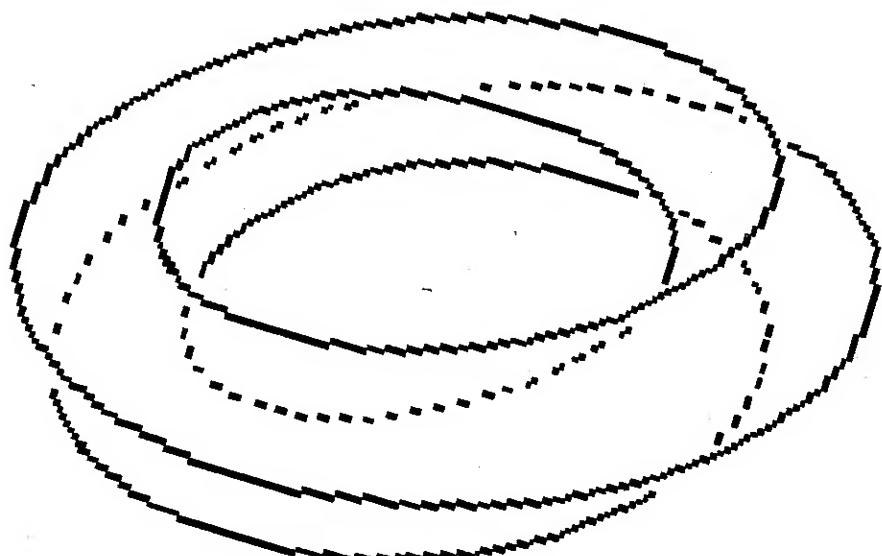
(\*) The model of a "black hole" is based on a handiwork from a solution of Einstein's equation, from Schwarzschild (1917), referring to an EMPTY region of the universe. We'll talk about it in a later album.

# THE MONOHEDRON

We can generate it by revolving a square around an axis contained within its plane and rotating it by  $\pi/2$  at each turn.



... or by thickening the Möbius ribbon.

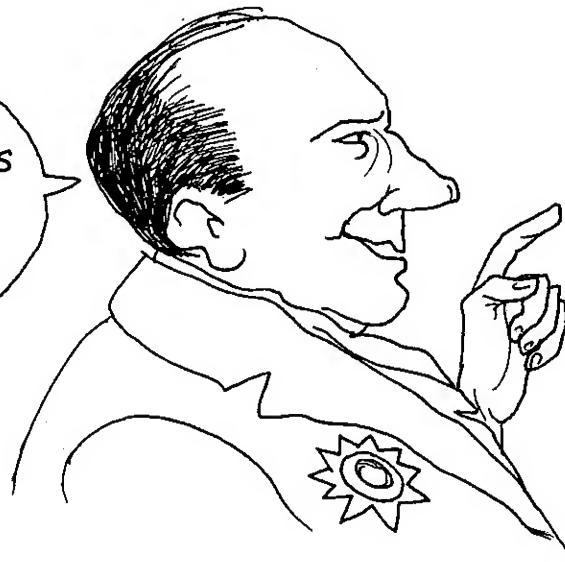


# APPENDIX 2

## SPACETIMES AND GROUPS

In 1850, Mikhail Valisevich Ostrogradsky to Bernhard Riemann

Listen my friend, why waste so much effort to explore those twisted spaces coming from your imagination since we are living in this stupidly euclidean space?

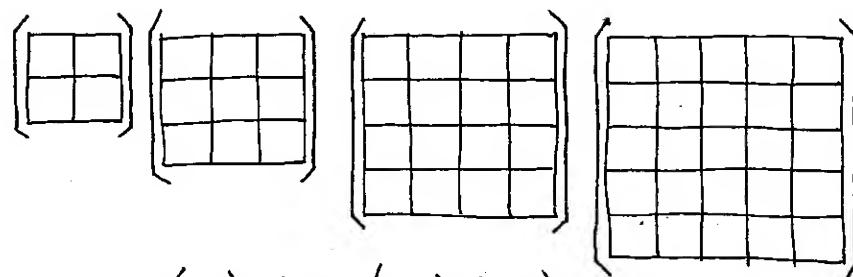


Time has passed.

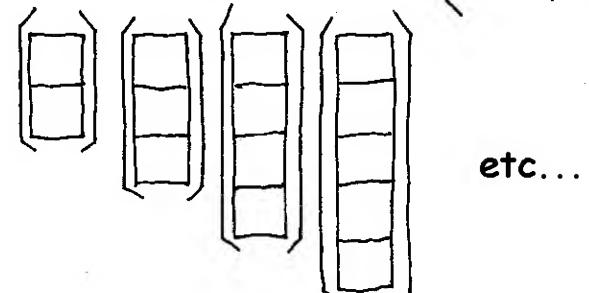
The permanent evolution of science shows that each time an advance occurred, it was done by abandoning some naive vision coming from our senses. Facts show that mathematicians, especially the geometers, always had a vision of things which revealed itself closer to the experiences of physicists and observations of astronomers than earlier visions which eventually fell into obsolescence. By manipulating new concepts with pencil and paper they create, perhaps without realizing it, the reality of tomorrow. To understand for example SPECIAL RELATIVITY, you must make an effort to do a real LET GO your vision of the world.

Are you ready to follow me ?

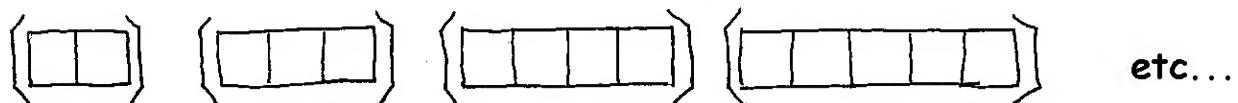
The letter M will designate a square MATRIX (n lines, n columns)



A COLUMN VECTOR is a matrix with n lines and 1 column :



A LINE VECTOR is a MATRIX with 1 line and n columns :



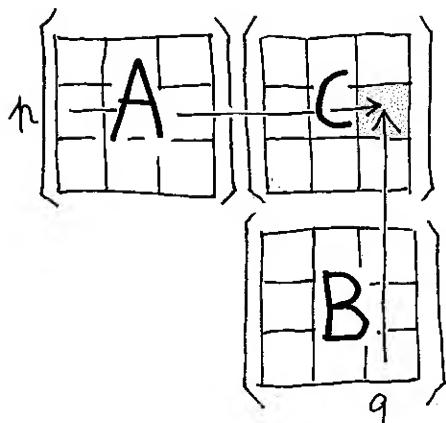
etc...

## MULTIPLICATION OF TWO SQUARE MATRICES WITH THE SAME FORMAT

(having the same number of lines = number of columns)

$$\begin{pmatrix} A \end{pmatrix} \times \begin{pmatrix} B \end{pmatrix} = \begin{pmatrix} C \end{pmatrix}$$

$$C = A \times B \quad \text{we multiply "LINES-COLUMNS"}$$



Mnemonic technique : we place the two matrices A and B of the product matrix  $A \times B$  as shown on the left and we multiply elements by elements, by adding the elements of the line  $p$  of the matrix A by the elements of the column  $q$  of the matrix B. This way we obtain the element on the  $p^{\text{th}}$  line and  $q^{\text{th}}$  column of the matrix  $C = A \times B$ .

FUNDAMENTAL: THIS PRODUCT IS NOT, IN GENERAL, COMMUTATIVE.

$$A \times B \neq B \times A !$$

## IDENTITY MATRIX $I$

For every set of square matrices with  $n$  lines,  $n$  columns [we say "of format  $(n,n)$ "] we associate an identity matrix, denoted by  $I$

$$\left( \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array} \right) \quad \left( \begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ \hline \end{array} \right) \quad \left( \begin{array}{|c|c|c|c|} \hline 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \\ \hline \end{array} \right) \quad \left( \begin{array}{|c|c|c|c|c|} \hline 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \\ \hline \end{array} \right)$$

etc...

We have:

$$A \times I = I \times A = A$$

## TRANSPOSE OF A MATRIX, DENOTED ${}^t A$

It is the symmetric inverse of the square table with respect to its MAIN DIAGONAL.

$${}^t \left( \begin{array}{|c|c|} \hline \end{array} \right) = \left( \begin{array}{|c|c|} \hline \end{array} \right)$$

$${}^t \left( \begin{array}{|c|c|c|} \hline \end{array} \right) = \left( \begin{array}{|c|c|c|} \hline \end{array} \right)$$

$${}^t \left( \begin{array}{|c|c|c|c|} \hline \end{array} \right) = \left( \begin{array}{|c|c|c|c|} \hline \end{array} \right)$$

etc...

$${}^t \left( \begin{array}{|c|c|c|c|} \hline \end{array} \right) = \left( \begin{array}{|c|c|c|c|} \hline \end{array} \right)$$

WE WILL POSE that the transpose of a vector, or column matrix :

$$X = \begin{pmatrix} | \\ | \\ | \\ | \end{pmatrix}$$

is the corresponding line matrix :

$${}^t X = \begin{pmatrix} \square & \square & \square & \square \end{pmatrix}$$

## MULTIPLICATION OF A LINE OR COLUMN MATRIX BY A SQUARE MATRIX

For the column matrix, MULTIPLY ON THE LEFT :

$$A \times X = \begin{pmatrix} \square & \square & \square & \square \end{pmatrix} \times \begin{pmatrix} | \\ | \\ | \\ | \end{pmatrix}$$

For the line matrix, MULTIPLY ON THE RIGHT :

$$A \times {}^t X = \begin{pmatrix} \square & \square & \square & \square \end{pmatrix} \times \begin{pmatrix} \square & \square & \square \end{pmatrix}$$

## PRODUCTS OF A COLUMN MATRIX $\rightleftharpoons$ AND OF A LINE MATRIX :

$$\begin{pmatrix} \square & \square & \square \end{pmatrix} \times \begin{pmatrix} | \\ | \\ | \\ | \end{pmatrix}$$

$$\begin{pmatrix} \square & \square & \square \end{pmatrix} \times \begin{pmatrix} \square & \square & \square \end{pmatrix}$$

${}^t X \times X =$  matrix with 1 line, 1 column = SCALAR

$X \times {}^t X =$  square matrix with format  $(n, n)$

So, a scalar is a matrix with only one line and one column ?

yep, when we are doing the groceries, we actually multiply and add matrices !

and no one ever told us that !

A COMPLEX NUMBER  $(a, b)$  or  $a + ib$  is really a square matrix :

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

And the imaginary number  $i$  is

$$i = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$
$$i \times i = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -1$$
$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Although MATRICES and MATRIX ALGEBRA are essential tools for the understanding of our physics and mathematics, the teaching of these subjects have fallen everywhere into ... obsolescence!

Square matrices can have an inverse, denoted  $A^{-1}$  such that :

$$A^{-1} \times A = A \times A^{-1} = I$$

A first theorem, without proof:

$$(A \times B)^{-1} = B^{-1} \times A^{-1}$$

A second theorem, without proof:

$${}^t(A \times B) = {}^tB \times {}^tA$$

the proofs are easy but without much interest (if you really want to...)

... with these tools, we will be able to reach the outposts of science

watch out, he's coming back!

but... that's not the right direction !?!

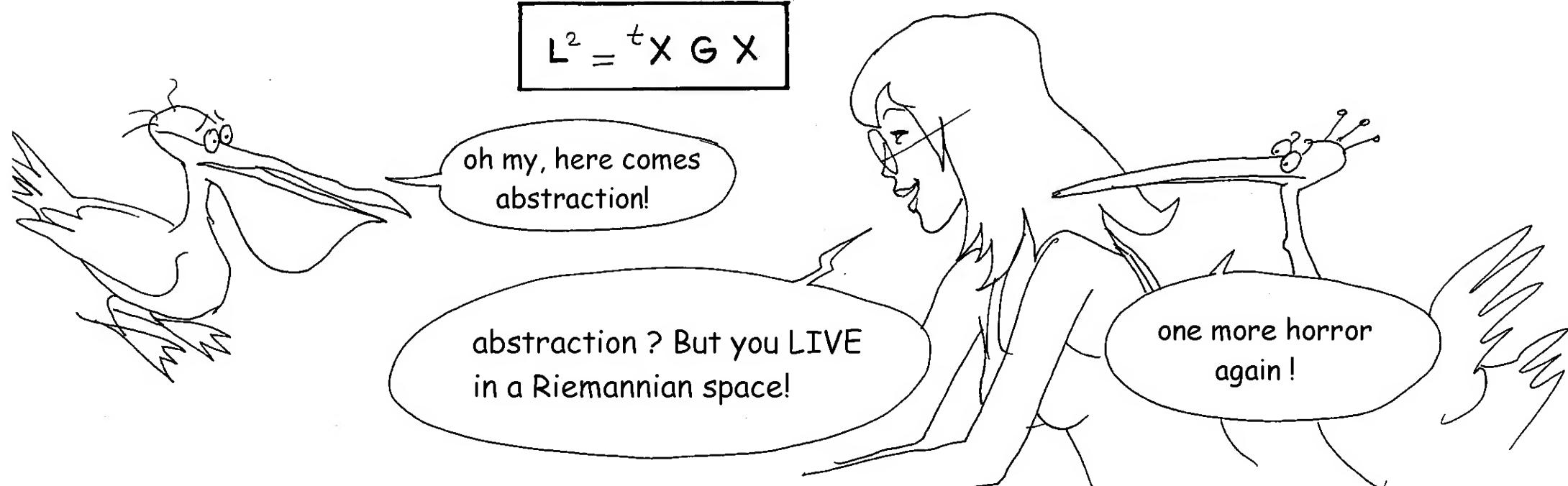
# RIEMANNIAN SPACES (\*)

We will call **GRAM MATRICES** square matrices in which all non-diagonal elements are zero and all elements of the **MAIN DIAGONAL** have value  $\pm 1$ .

$$\left( \begin{matrix} \pm 1 & 0 \\ 0 & \pm 1 \end{matrix} \right) \left( \begin{matrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 \end{matrix} \right) \left( \begin{matrix} \pm 1 & 0 & 0 & 0 \\ 0 & \pm 1 & 0 & 0 \\ 0 & 0 & \pm 1 & 0 \\ 0 & 0 & 0 & \pm 1 \end{matrix} \right) \left( \begin{matrix} \pm 1 & 0 & 0 & 0 & 0 \\ 0 & \pm 1 & 0 & 0 & 0 \\ 0 & 0 & \pm 1 & 0 & 0 \\ 0 & 0 & 0 & \pm 1 & 0 \\ 0 & 0 & 0 & 0 & \pm 1 \end{matrix} \right) \dots$$

Let a vector  $X$  belonging to a space  $\mathcal{E}$  with  $n$  dimensions. We will say that this space is Riemannian if the square of the length of the vector  $X$  is defined by :

$$L^2 = {}^t X G X$$



(\*) Mathematicians are not all in agreement on the terminology. Here we decide to regroup under this name all spaces having a  $\pm 1$  signature.

Think about it. The identity matrix of format (3,3) is a particular Gram matrix

$$\mathbf{I} = \begin{pmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & +1 \end{pmatrix}$$

Yes and so what?

Let  $\mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$  then  ${}^t \mathbf{X} = [x, y, z]$

$$\text{and } L^2 = {}^t \mathbf{X} \mathbf{I} \mathbf{X} = {}^t \mathbf{X} \mathbf{X} = x^2 + y^2 + z^2$$

which is the square of the

$$\text{EUCLIDEAN LENGTH } L = \sqrt{x^2 + y^2 + z^2}$$

# SIGNATURE

The signature of these spaces is the sequence of signs of the Gram metric. In the case of three dimensional euclidean space it's: (+ + +)

In a two dimensional space, the Gram matrix corresponding to an euclidean space would be  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  and the signature (+ +)

We now ask the following question: is there a set of matrices  $\mathbf{M}$  acting on the vector

$$\mathbf{X} = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and which preserve its length?}$$

We will now formally make the calculation in the most general case, which is the case of a Riemannian space with  $n$  dimensions defined by its Gram matrix  $G$

Let  $M$  be a matrix acting on the vector  $X$  by transforming it into the vector:

$$X' = MX$$

The square of the length, of the norm of vector  $X'$  is

$$L'^2 = {}^t X' G X' = {}^t (MX) G (MX) = ({}^t X {}^t M) G (MX) = {}^t X ({}^t M G M) X$$

the lengths  $L'$  and  $L$  will be equal if :

$${}^t M G M = G$$

Let's apply this to an euclidean space of  $n$  dimensions :

$${}^t M M = I$$

Which simply means that :

$$M^{-1} = {}^t M$$

These matrices are said to be orthogonal matrices. We will show it in the two dimensional case

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$a^2 + b^2 = 1 \quad ; \quad c^2 + d^2 = 1 \quad ; \quad ac + bd = 0$$

We look for the matrices  $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  which satisfy these relations.

These matrices M form a set ... 

We will see that these matrices also form a ...

# GROUP

Here's the magical word of physics coming out ! But what is a group ?

It is a set of tricks which act on a set of things... In our case the tricks are MATRICES and the things are points, or set of points of a space.

Souriau has an habit to say:

- A group is made to transport.
- The manner that we transport is more important that what is being transported.

In the comic book we have read "tell me how you move, I will tell you WHAT you are".

Here we could say:

Tell me how you let yourself being transported and I will tell you in which family of geometrical beings you belong. In short, in which space you inhabit.

Hence the close relationship GROUP  $\Leftrightarrow$  GEOMETRY

The axioms that define a group were introduced by Norwegian Sophus Lie. We also call group of matrices LIE GROUPS. Now let's look at the axioms:

Consider a set of things acting on each other. Let's call them  $\alpha, \beta, \gamma \dots$

They form a set  $\Sigma$

We can compose them through a LAW OF COMPOSITION that we will write  $\gamma = \alpha \circ \beta$

1: If  $\alpha$  and  $\beta$  belongs to a set,  $\alpha \circ \beta$  also belongs the set. We say that this law of composition is closed under group  $\Sigma$ . (dogs do not make cats)

2: There exists an element  $e$ , called UNIT ELEMENT such as for all element  $\alpha$  of the group, we have

$$e \circ \alpha = \alpha \circ e = \alpha$$

3: Every element  $\alpha$  has a INVERSE denoted  $\alpha^{-1}$  such as :

$$\alpha \circ \alpha^{-1} = e$$

4: The composition operation is associative, meaning that:

$$(\alpha \circ \beta) \circ \gamma = \alpha \circ (\beta \circ \gamma)$$

we will almost NEVER use the fourth axiom. In fact it is very difficult to find operation of composition which are NOT ASSOCIATIVE.

The physicist will ONLY work on GROUP OF MATRICES also called LIE GROUP.

- We will have sets of square matrices  $M$ ,
- The composition operation  $\circ$  will be the NON-COMMUTATIVE MATRIX PRODUCT  $M_1 \times M_2$
- The unit element  $e$  will be systematically the identity matrix  $I$  in the considered format  $(n,n)$

# DISCRETE GROUPS

We call discrete groups those groups forming sets of finite elements. (here of matrices)  
Gram matrices of format  $(2,2)$  form a group of 4 elements.

$$g = \begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix} \quad \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right\}$$

Incidentally, these matrices are identical to their inverse. What do they represent ?

Let them ACT on vectors  $X = \begin{pmatrix} x \\ y \end{pmatrix}$  of a 2D space:

$$\left\{ \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} X \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix} \right. \text{ symmetry with respect to the oy axis.}$$

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} X \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix} \right. \text{ symmetry with respect to the ox axis.}$$

$$\left\{ \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} X \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix} \right. \text{ symmetry with respect to the origin.}$$

Our conditions are met:  
these symmetries conserve  
length.

# GROUP WITH 1 (or many) PARAMETERS

The matrices  $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$  satisfy our criteria and constitute the group of rotations of the plane around the origin.

It's a group with 1 parameter. (the angle  $\theta$ )

Up to now, I'm still following.  
It does look simple, doesn't it ?

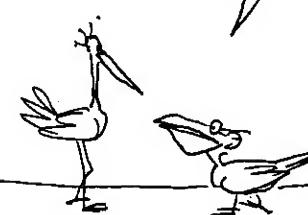
The number of parameters is called the **DIMENSION OF THE GROUP**, but it has nothing to do with the dimension of the space on which it is **ACTING**.



Maybe but with the author  
I am wary... It starts simple enough,  
but suddenly he'll make you smoke  
your neurons badly...

Beyond some level of deep thinking  
the brain should be wired to a fuse!

Myself, I've never fully recovered  
from TOPO THE WORLD ...



The matrices

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

form a group named  $SO(2)$ , for "special orthogonal"

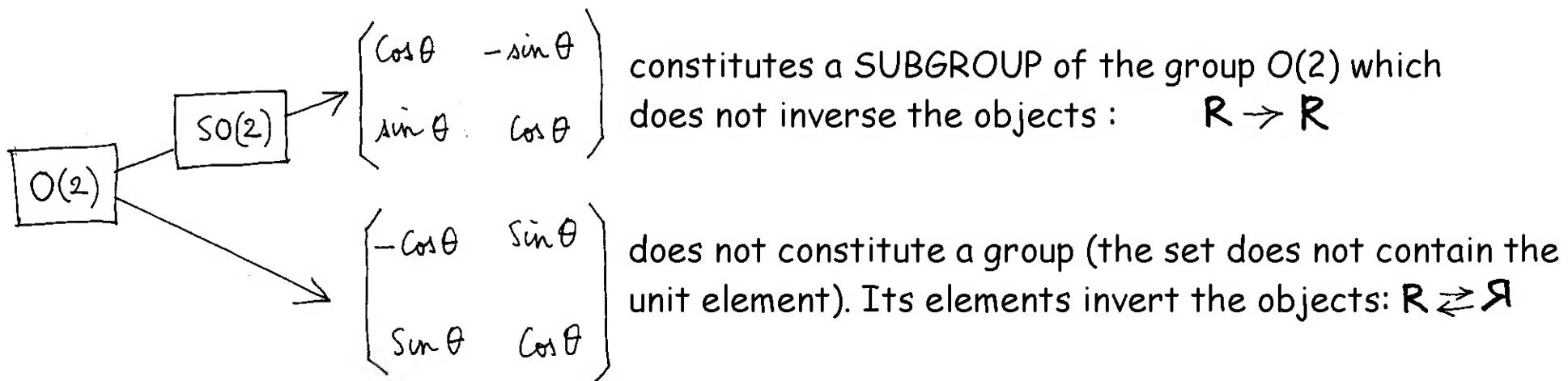
# ORIENTATION

By multiplying this matrix by one of the two matrices inverting the objects ( $R \rightleftharpoons R$ ) for example the one which apply a symmetry with respect to the oy axis, we obtain:

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} -\cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

note that when  $\theta = \pi$ , we have a symmetry with respect to the ox axis.

We get a second set of matrices which are also orthogonal matrices since they satisfy  ${}^T MM = I$ . The union of these two sets constitute the ORTHOGONAL GROUP  $O(2)$ . The element of this group will be denoted  $a$  and we will say that this group has TWO COMPONENTS.



# ISOMETRY GROUP

The set of actions conserving lengths in a two dimensional space includes:

- Rotations
- Symmetries
- Translations

which can be expressed with matrices:

$$\begin{array}{c} \text{SE}(2) \xrightarrow{\quad} \begin{pmatrix} \cos\theta & -\sin\theta & \Delta x \\ \sin\theta & \cos\theta & \Delta y \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x \cos\theta - y \sin\theta + \Delta x \\ x \sin\theta + y \cos\theta + \Delta y \\ 1 \end{pmatrix} \xrightarrow{\quad} \text{R} \rightarrow \text{R} \\ \text{E}(2) \xrightarrow{\quad} \begin{pmatrix} \cos\theta & \sin\theta & \Delta x \\ \sin\theta & \cos\theta & \Delta y \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} -x \cos\theta + y \sin\theta + \Delta y \\ x \sin\theta + y \cos\theta + \Delta y \\ 1 \end{pmatrix} \xrightarrow{\quad} \text{R} \rightleftharpoons \text{R} \end{array}$$

We obtain the 2D EUCLIDEAN GROUP E(2) which is the ISOMETRY GROUP of the EUCLIDEAN SPACE in TWO DIMENSIONS. Its first COMPONENT SE(2) ("Special Euclid 2d") is a SUBGROUP. The second component is a set of matrices WHICH INVERT OBJECTS, but does not constitute a group.

In 2D it is possible to completely do the calculations explicitly. What has been done in 2D can be extended to 3D. The Gram matrix is the 3D identity matrix.

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

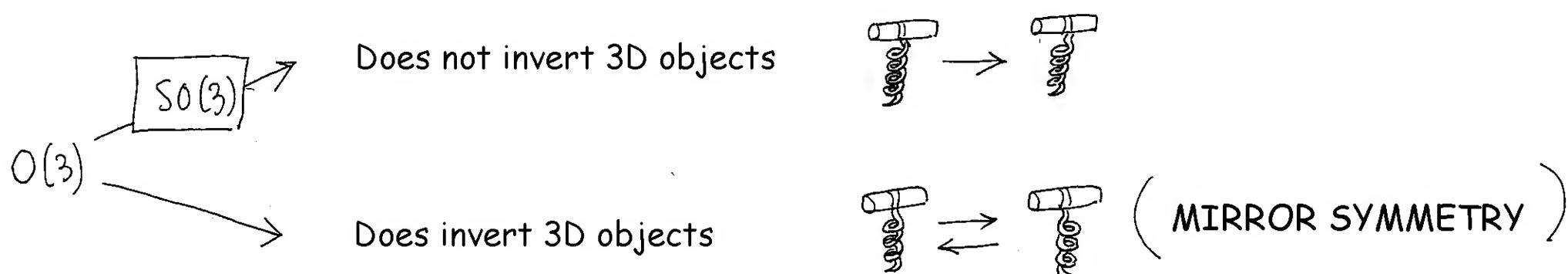
The square of the length is:  $L^2 = {}^t X I X$  the signature: (+ + +)

Let a matrix  $M$  acting on the vector  $X$  such that:  $X = MX'$

The conservation of the length leads to  $L'^2 = {}^t X' I X' = {}^t (MX)(MX) = {}^t X ({}^t M M) X$   
 $L' = L$  if:

$${}^t M M = I \quad \text{or} \quad M^{-1} = {}^t M$$

The matrices having this property, which are square matrices (3,3), are said to be ORTHOGONALS and constitute the ORTHOGONAL GROUP  $O(3)$ , which has TWO COMPONENTS:



By adding the translation vector

$$c = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

We construct the 3D euclidean group  $E(3)$  which inherits from properties of the orthogonal group  $O(3)$  on top of which it is constructed. We will call  $a$  the element coming for  $O(3)$  and we will write :

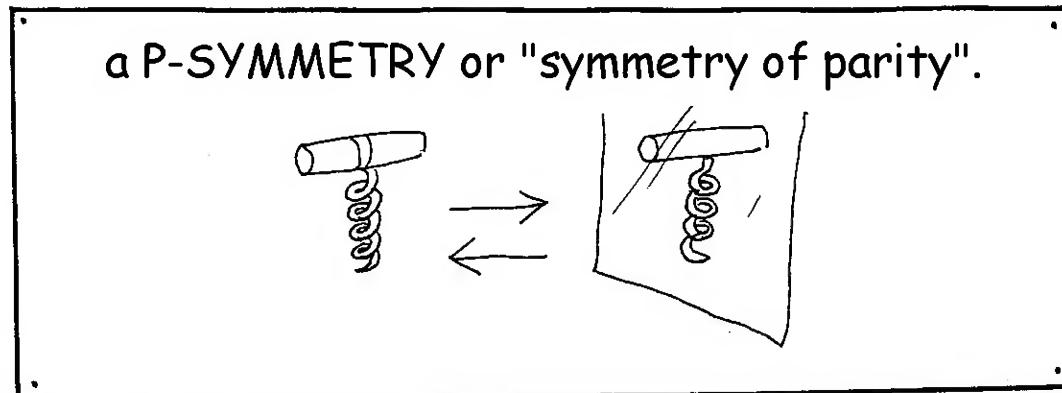
$$O = \begin{pmatrix} a & c \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & & & \Delta x \\ & (3,3) & & \Delta y \\ & & & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ acting on } \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

This ACTION, written in matrix form, allowing elements from the 3D euclidean group  $E(3)$  to act on the vector  $X$ , differs from the usual matrix multiplication of the like

$$X' = M X$$

which is just a form of ACTION among others. The concept of action is essential and we will reuse it later on.

Half the matrices forming the euclidean group transform oriented objects (the cork-screw) in their mirror image. We will say that they operate



## WHEN MATHEMATICIANS INVENT MIRRORS

Here for a few steps the mathematician precedes the physicist. After practicing rotations and translations, the mathematician invents the group notion, Gram matrices, construct the  $SE(3)$  subgroup which does not invert the objects by PHYSICALLY TRANSPORTING them. But the group produces elements that a simple physical transport could not create. By combining rotations and translations we will never create a "left-handed" cork-screw from a "right-handed" one. But the complete group predicts the "existence" of such ENANTIOMORPHIC objects living "on the other side of the mirror".



so we think we inhabit in an ELLIPTICAL RIEMANNIAN space or 3-D EUCLIDEAN SPACE, with signature (+ + +) which gives us among others PYTHAGORA'S THEOREM. But what about the spaces with signature ( - - - ) ?

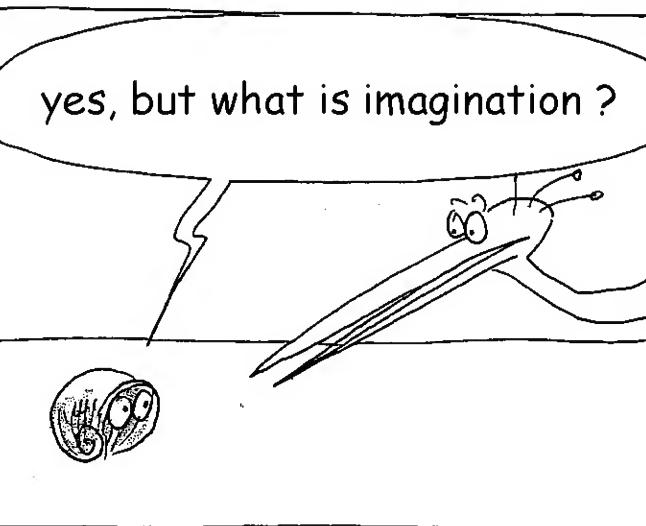
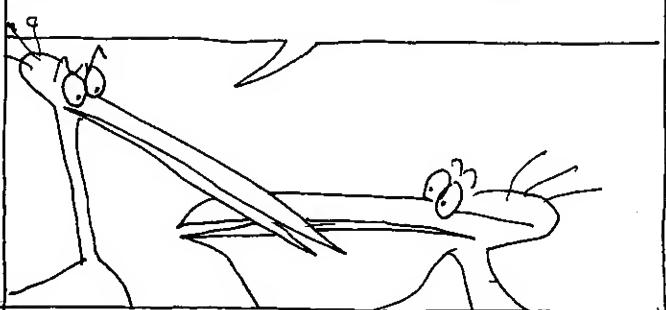


We call them IMPROPER EUCLIDEAN spaces. Their lengths are PURE IMAGINARY:

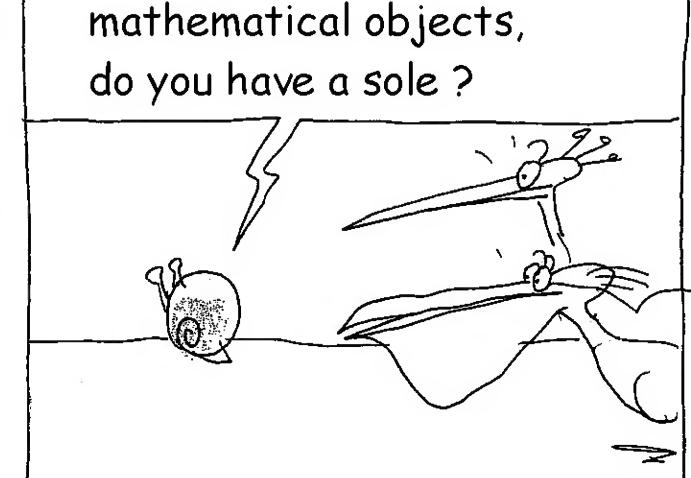
$$L = \sqrt{-x^2 - y^2 - z^2}$$

We'll come back at the end of all this on strange spacetimes where time is pure imaginary.

OK, let's not exaggerate.  
A pure imaginary time can only  
be a product of the imagination.

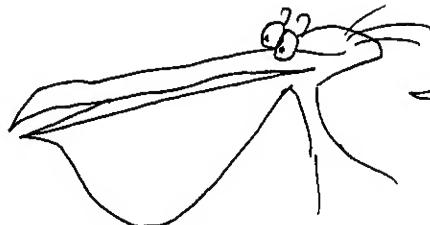


mathematical objects,  
do you have a sole ?



# HYPERBOLIC RIEMANNIAN SPACES

These are spaces which have both + signs and - signs in their signature. The emergence of the SPECIAL THEORY OF RELATIVITY consisted simply in realizing that instead of living in an Euclidean space of signature (+ + +) : a 3D HYPERSURFACE perpendicular to time, we lived in a hyperbolic Riemannian space, with signature (+ - - -), MINKOWSKI'S SPACE.



Tiresias, how can you say such horrors ?

The GRAM matrix is then

$$G = \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{Bmatrix}$$

Let's change letter to designate a space-time vector :

$$\xi = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

We'll define a space-time translation vector which we'll write :

$$C = \Delta \xi = \begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

We'll consider infinitesimal vectors :

$$d \xi = \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}$$

We will obtain (by taking  $c$ , the speed of light, = 1) the infinitesimal length :

$$ds^2 = {}^t d\mathbf{g} G d\mathbf{g} = dt^2 - dx^2 - dy^2 - dz^2$$

which we'll call MINKOWSKI'S METRIC and we can write it with a simple change of variables:

$$c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

We will proceed like we did for the euclidean group and the euclidean space. We will start by considering a 2D space-time:

$$\eta = \begin{pmatrix} t \\ x \end{pmatrix}$$

where the element of length, its 2D metric is  $ds^2 = {}^t d\eta G d\eta$   
with, as Gram's metric :

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

We will construct the ISOMETRY GROUP of this space...

we will proceed like we did for the euclidean space. We'll set aside for a moment the presentation under the differential form. We are looking for a group of matrices  $L$ , acting on the vector  $\xi$  according to:

$$\xi' = L \xi$$

which preserves this strange "hyperbolic length", meaning such that:

$$L'^2 = \xi' G \xi' = \xi^T (L^T G L) \xi = \xi^T (L^T G L) \xi = L^T G L = L^2 = \xi^T G \xi \text{ si :}$$

$$\boxed{L^T G L = G}$$

In 4D those are matrices with 4 lines, 4 columns (of format (4,4)). The above formula is the definition of the LORENTZ group (of matrices). To show it explicitly, we will limit ourselves to a 2D space-time (t,x)

$$L = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\text{giving } a^2 - c^2 = 1 \quad ; \quad b^2 - d^2 = 1 \quad ; \quad ab - cd = 0$$

which gives us a first

$$\begin{bmatrix} \cosh \eta & \sinh \eta \\ \sinh \eta & \cosh \eta \end{bmatrix}$$

$$\text{since } \cosh^2 \eta - \sinh^2 \eta = 1$$

⇒ The trigonometric lines are replaced by hyperbolic lines

$$\left\{ \begin{array}{l} \text{ch} \eta = \frac{e^\eta + e^{-\eta}}{2} \\ \text{sh} \eta = \frac{e^\eta - e^{-\eta}}{2i} \end{array} \right\} \left\{ \begin{array}{l} \cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \\ \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i} \end{array} \right\} z = e^{i\theta} = \cos \theta + i \sin \theta$$

The LORENTZ GROUP is the equivalent of the rotations in MINKOWSKI's space.

## DISCRETE GROUP

The 2D Gram matrices are Lorentz matrices, satisfying

$${}^t L G L = G$$

${}^t G G G = G$  with  $GG = I$  and  ${}^t G = G$ , so in 2D we have the discrete group :

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right\}$$

We will get the complete Lorentz group, with four components:

$\begin{bmatrix} \text{ch} \eta & \text{sh} \eta \\ \text{sh} \eta & \text{ch} \eta \end{bmatrix}$	$\begin{bmatrix} \text{ch} \eta & -\text{sh} \eta \\ \text{sh} \eta & -\text{ch} \eta \end{bmatrix}$	$\begin{bmatrix} -\text{ch} \eta & \text{sh} \eta \\ -\text{sh} \eta & \text{ch} \eta \end{bmatrix}$	$\begin{bmatrix} -\text{ch} \eta & -\text{sh} \eta \\ -\text{sh} \eta & -\text{ch} \eta \end{bmatrix}$
--	--	--	--

$\underbrace{\hspace{100px}}$

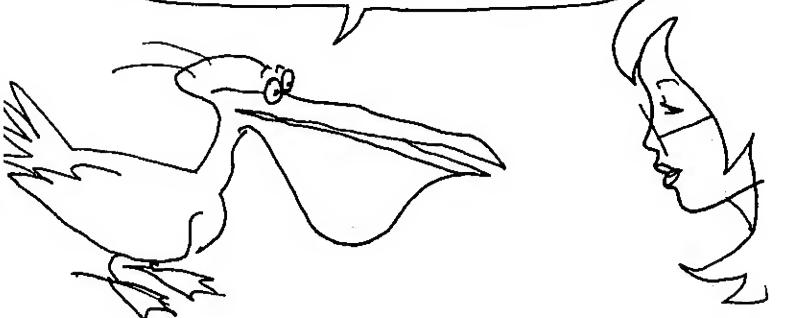
orthochron subgroup

$\underbrace{\hspace{100px}}$

antichron subset

# SPECIAL RELATIVITY

We talked about SPECIAL RELATIVITY. But what is Einstein's theory ?



go back to the length's calculation in this hyperbolic Riemann space that is the MINKOWSKI'S SPACE in differential form, given by the metric:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

This means that our MOVEMENTS ARE WRITTEN (\*) on a 4D hypersurface.  $(x, y, z, t)$  are COORDINATES on it. In FASTER THAN LIGHT we explain that the inscribing of a coordinates system on this hypersurface corresponds to the observation done by the

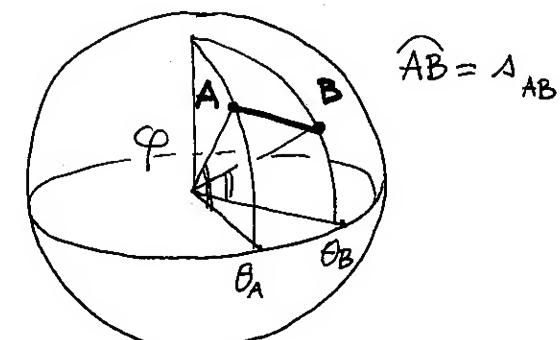
PHYSICIST of this hypersurface where the only INTRINSIC value is the length  $S$ . There is the same relation between these coordinates and this length  $S$ , which is measured in METERS and which is converted in PROPER TIME  $\tau$  using the relation  $ds = cdt$  where  $c$  is the characteristic speed only between the coordinates of longitude  $\theta$  and latitude  $\varphi$  used for finding points on a sphere and the length of the trajectory  $\widehat{AB}$ . What is shown by this formula is that when we take coordinates  $(x, y, z, t)$ , we can deduce a speed

$$V = \frac{\sqrt{dx^2 + dy^2 + dz^2}}{dt}$$

For the time  $d\tau$  to remain real, we must have  $V < c$   
the limit movement will correspond to  $V = c$ , and then  $d\tau = 0$

⇒ the proper time of the PHOTON is "frozen"

(\*) in Arabic: MEKTOUB



For particles traveling at  $V < c$  we have LORENTZ'S CONTRACTION applying

$$c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \Rightarrow \frac{d\tau}{dt} = \sqrt{1 - \frac{V^2}{c^2}}$$

$\tau$  is the time showing on the traveller's watch moving at velocity  $V$ , which is illustrated in the album **EVERYTHING IS RELATIVE**. And when  $V$  approaches  $c$  "time is freezing in the chronometers". But let's come back to LORENTZ'S GROUP. Its elements act on a series of points of spacetime which constitute a MOVEMENT. By letting an element  $L$  of the Lorentz group on a given movement we obtain another movement. The fact the group contains ANTICHRONS elements shows that these TIME-REVERSED movements have to be taken into consideration. For example, here's a matrix which belongs to the Lorentz group:

$$L = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^t L G L = G \quad \text{with} \quad G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The action is:

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -t \\ x \\ y \\ z \end{pmatrix} \quad \text{TIME INVERSION}$$

When we defined the ORTHOGONAL GROUP, subgroup of the isometry group of EUCLIDEAN SPACE, we completed it with the SPATIAL TRANSLATIONS vector

$$c = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

by constructing the EUCLIDEAN GROUP, its isometry group

element of orthogonal group  $O(3)$

$$\begin{pmatrix} a & c \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} r \\ 1 \end{pmatrix}$$

$$r = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Similarly, from LORENTZ'S GROUP we are going to construct the POINCARÉ GROUP, the isometry group of MINKOWSKI'S SPACE.

$$c = \begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

spacetime translations

$$\begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} \xi \\ 1 \end{pmatrix}$$

$$\xi = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

The Poincaré group, through its subgroup  $\begin{pmatrix} L & 0 \\ 0 & 1 \end{pmatrix}$  inherits of the properties of the Lorentz group and likewise has four components:

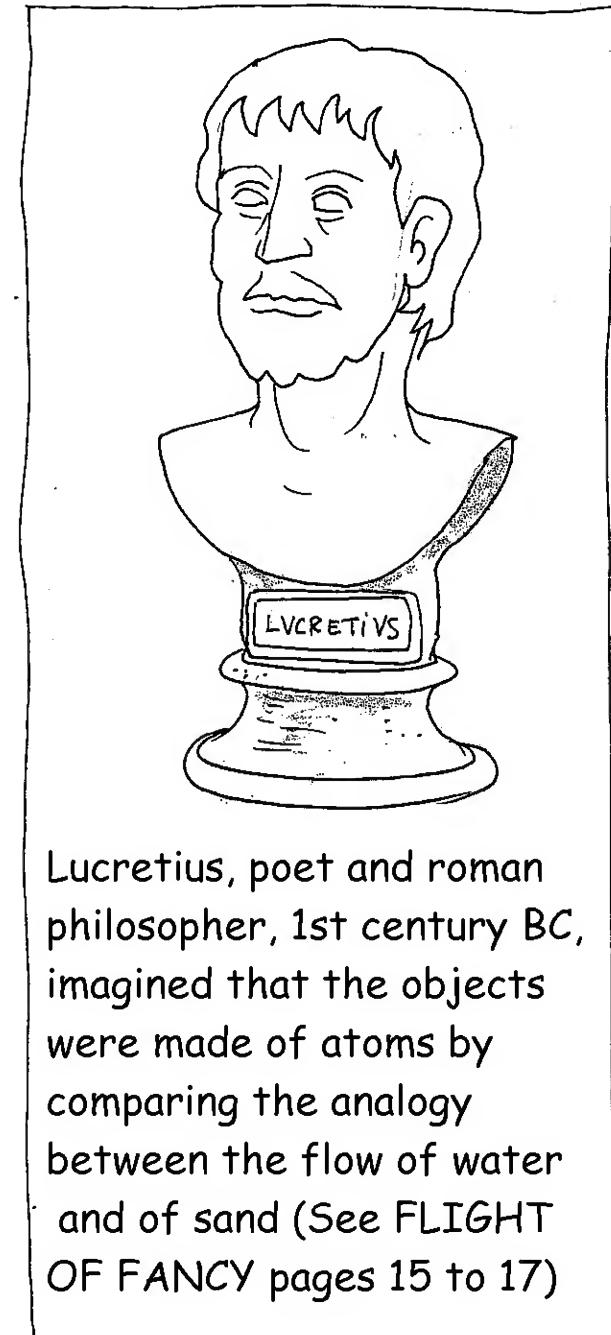
- TWO ORTHOCHRONS (not inverting time)
- TWO ANTICHRONS (inverting time)

We still have to understand the PHYSICAL SIGNIFICANCE of this temporal inversion

# SPACE, GROUPS AND OBJECTS

We started from the euclidean space and limited ourselves to 2D so that we could show the calculations explicitly. We then construct its ISOMETRY GROUP, the EUCLIDEAN GROUP. This group goes along the euclidean space and can ACT on the objects, points living in this space. But we can take the problem backward: take a group, as an abstract object, purely mathematical, allowing to envisions ACTIONS and discover the "space that goes along", the only one where these actions can be realized - "the matching space" in other words. Hence space and its (isometry) group mutually gives themselves their existences.

But there is more - the group generate the OBJECTS of the space to which it is linked by the INVARIANCES OF THE ACTIONS OF A SUBGROUP. Let's give an example: the rotations around a point in 2D euclidean space constitute one of its subgroups. The invariants objects are then the family of circles centered on this point. This is how, in terms of groups, that we define the circle!



Lucretius, poet and roman philosopher, 1st century BC, imagined that the objects were made of atoms by comparing the analogy between the flow of water and of sand (See FLIGHT OF FANCY pages 15 to 17)

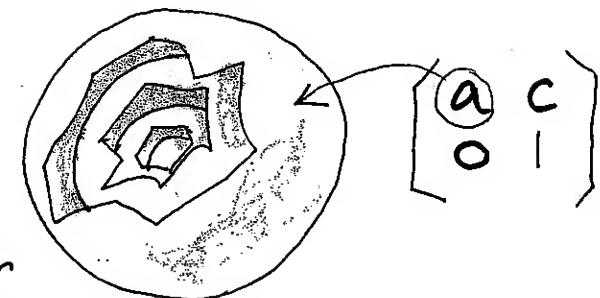
In the 3D euclidean group, the rotations around a point also constitute one of its subgroups. What are the objects that these **ACTIONS OF THE SUBGROUP** leaves **INVARIANT**?

Answer: the family of **SPHERES** centered on that point. The concept of **INVARIANT** by such or such action of the group or one of its subgroups is a fundamental concept of **GROUP THEORY**.

In the euclidean group, where time is absent, the group generates itself the **OBJECTS** which will populate the space to which it is associated.

When time enters the picture, the group becomes a **DYNAMIC GROUP**. It no longer manages static objects, but **SET OF "EVENT-POINTS"** that we can name **TRAJECTORIES** or **MOVEMENTS**. At the beginning of the 20th century, the remarkable German mathematician Emmy Noether (qualified by Einstein as "movement of physics") gave her name to one of the most important theorem of physics that says that for every subgroup of a dynamic group corresponds an **INVARIANT**.

In the **POINCARÉ GROUP** we find the **SUBGROUP OF TIME TRANSLATIONS**, represented by the matrix on the right. Group with 1 parameter, there is a corresponding invariant, a scalar: the **ENERGY E**. This is how, in terms of groups, that we define energy!



$$\begin{pmatrix} 1 & 0 & 0 & 0 & \Delta t \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} t \\ x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} t + \Delta t \\ x \\ y \\ z \\ 1 \end{pmatrix}$$

Second subgroup: the subgroup of SPATIAL TRANSLATIONS (matrix on the right), group with three parameters ( $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ ).

A new invariant corresponds to this subgroup:

the MOMENTUM

$$\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

this is how, with the help of DYNAMIC GROUPS that we define momentum. In that way, the quantifiable values of physics become GEOMETRICAL OBJECTS and this process of GEOMETRISATION OF PHYSICS constitute of the the pillars of MODERN PHYSICS.

By continuing playing that little game we could consider the subgroup of SPACETIME TRANSLATIONS

(matrix on the right)

The invariant object would then be the MOMENTUM-ENERGY FOUR-VECTOR.

$$\begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta x \\ 0 & 0 & 1 & 0 & \Delta y \\ 0 & 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} t \\ x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} t \\ x + \Delta x \\ y + \Delta y \\ z + \Delta z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \Delta t \\ 0 & 1 & 0 & 0 & \Delta x \\ 0 & 0 & 1 & 0 & \Delta y \\ 0 & 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} t \\ x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} t + \Delta t \\ x + \Delta x \\ y + \Delta y \\ z + \Delta z \\ 1 \end{pmatrix}$$

What is the use of QUANTIFIABLE VALUES IN PHYSICS ? Good question.

Answer = WE CAN ADD THEM UP !

The Poincaré group depends on ten parameters (we say that it has "ten dimensions" in simple math nerd terminology). There are 3 for spatial translations, 1 for temporal dimension. There remains six, which represent the dimension of the LORENTZ GROUP, which manage "spacetime rotations". If we consider the Lorentz group as a subgroup of the Poincaré group :

Noether's theorem says that it must have a corresponding "object" defined by six parameters which will be invariant under the action of this subgroup.

$$\begin{pmatrix} L & 0 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} \xi \\ 1 \end{pmatrix} = \begin{pmatrix} L\xi \\ 1 \end{pmatrix} \text{ with } \xi = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

In this object, SPIN is hiding. Souriau showed in 1972 its PURELY GEOMETRIC nature. It has the dimension of angular momentum. Now the Poincaré group manages the movements of the RELATIVISTIC MATERIAL POINT. The interpretation of spin as a purely geometric object is preferable.

## The "MOMENT"

The subgroups correspond to a kind of "dismantling of the group, part by part". When we do the opposite operation, we reconstitute the group. The set of invariants found earlier constitute what Souriau has called the "moment"

$$\text{moment} = \{ E, p_x, p_y, p_z, \dots \text{SPIN} \}$$

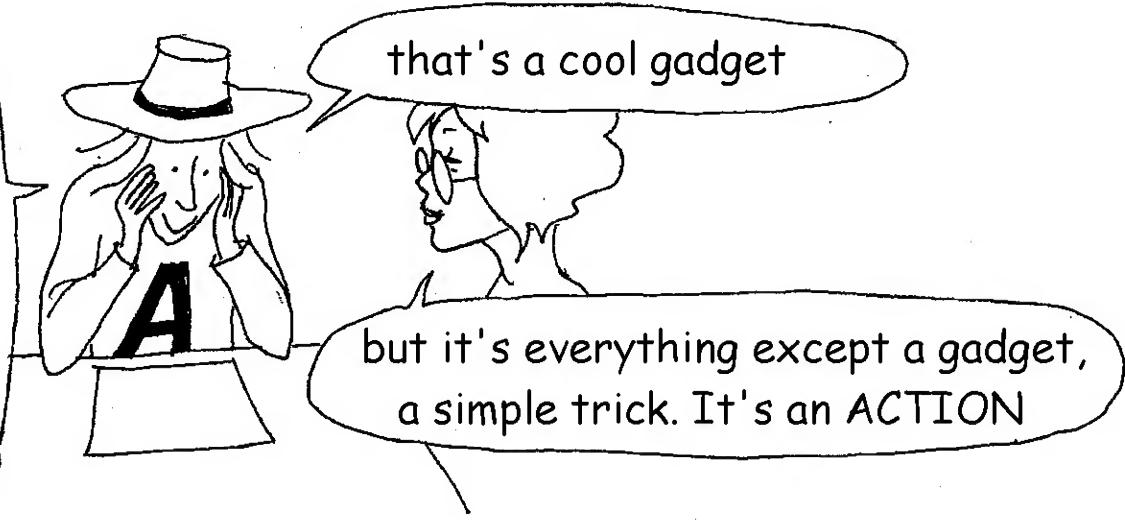
# ACTIONS OF A GROUP

I knew matrix multiplication:  $X' = MX$ , but I did not know this way to let a group of matrices ACT in a way to manage, for example in the euclidean group, rotations, symmetries and translations at one fell swoop.

$$X' = \begin{bmatrix} a & c \\ 0 & 1 \end{bmatrix} \times X = \begin{bmatrix} aX + c \\ 1 \end{bmatrix}$$

but... there are not so many ways to make a GROUP ACT. There is this one, and that's it, no ?

there is already one that you forgot!



the action of an element  $g$  of the group on another element  $g'$

$$g \times g' = g''$$

this makes it two

so what is a GROUP ACTION ?

A group can ACT on the elements of a set  $U$  and its ACTIONS are defined as follows :

Let  $g$  be the element of the group

Let  $\circ$  be the operation of composition

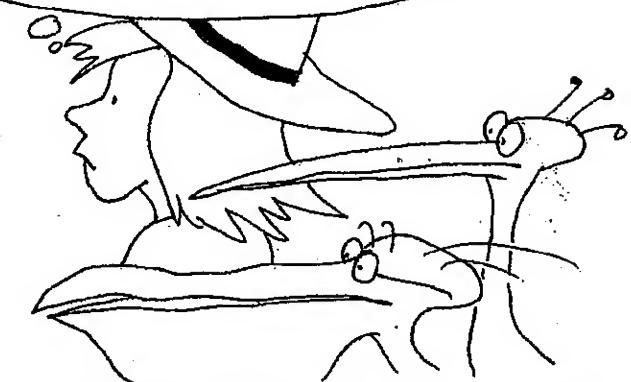
Let  $u$  be the element of the set  $U$

$A_g(u)$  will be an action of  $g$  on  $U$  if

$$A_g''(u) = A_g[A_g(u)]$$



it looks more or less like some transitive stuff...



If the action is simply the operation of composition  $\circ$   
 $g \circ (g' \circ u) = (g \circ g') \circ u = g'' \circ u$ , it works.

So the operation of composition is an action



I'm glad to learn it.  
But we are smashing open doors, no ?

Let's try with :

$$A_g(x) = \begin{bmatrix} a' & c' \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} a'x + c' \\ 1 \end{bmatrix}$$

which transforms  $X$  in  $X' = a'x + c'$



and then just reapply this

and what ?

I write  $Ag(X) = \begin{pmatrix} a & c \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} a'X + c' \\ 1 \end{pmatrix} = \begin{pmatrix} aa'X + ac' + c \\ 1 \end{pmatrix}$

and now, I'm lost, I don't recognize anything...



no everything's OK. Just do the product of the two matrices:

$$\begin{pmatrix} a & c \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} a' & c' \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} aa' & ac' + c \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a'' & c'' \\ 0 & 1 \end{pmatrix}$$

What you obtained is  $\begin{pmatrix} a'' & c'' \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} X \\ 1 \end{pmatrix}$  so:

$Ag[Ag'(X)]$  gives you  $Ag''(X)$  with  $g'' = g \times g'$

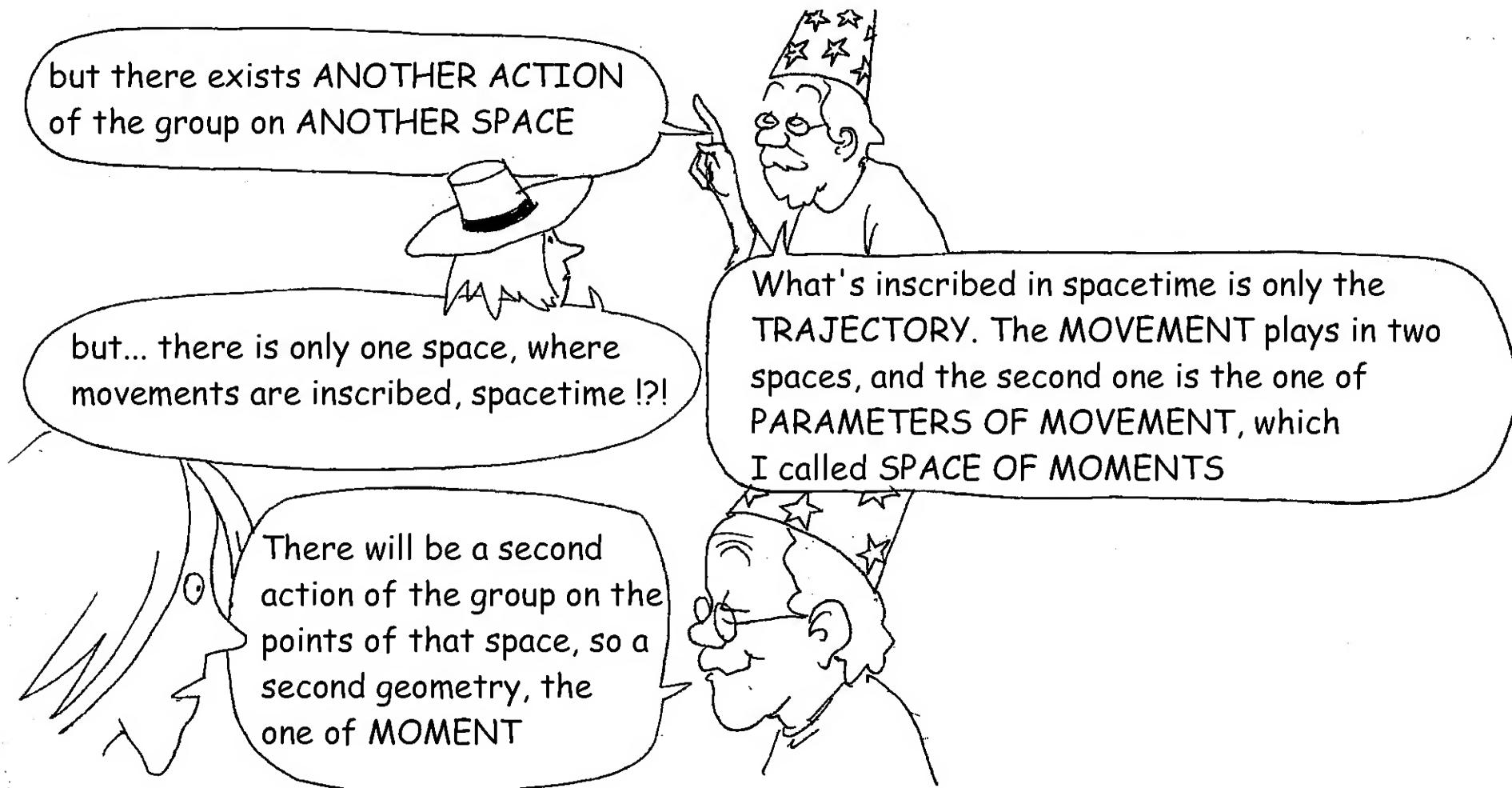
This means that  $\begin{pmatrix} a & c \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} X \\ 1 \end{pmatrix}$  is really an ACTION of an

element  $g$  of the euclidean group on the points  $X$  of the space.



and, in the same way  $\begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} \xi \\ 1 \end{pmatrix} = \begin{pmatrix} L\xi + C \\ 1 \end{pmatrix}$  with  $\xi = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$  is also an ACTION of the POINCARÉ GROUP on the "event-points"  $\xi$  of the SPACETIME

# BEWARE A GEOMETRY CAN HIDE ANOTHER!





$$J' = g \times J \times {}^t g$$

where  $J$  is an ANTSYMMETRIC matrix

we can verify that it is indeed an ACTION

$$Ag[Ag'(J)] = g \times [g' \times J \times {}^t g'] \times {}^t g = gg' J {}^t g' g$$

but  ${}^t[AB] = {}^t B {}^t A$  then  ${}^t g' {}^t g = {}^t (gg')$  and if  $g'' = gg'$

$$Ag[Ag'(J)] = g'' \quad {}^t g'' = Ag''(J)$$

The  $J$  matrix necessarily has the same format (5,5) of the  $g$  matrices of the group. In an antisymmetric matrix, the symmetric elements with respect to the main diagonal have opposite signs. The elements of the main diagonal are equal to zero (which is its own opposite). We can now count the components of this matrix

$$\begin{bmatrix} 0 & \ell \\ -\ell & 0 \end{bmatrix}$$

(2,2)

$$\begin{bmatrix} 0 & -\ell_z & -\ell_y \\ \ell_z & 0 & -\ell_x \\ -\ell_y & \ell_x & 0 \end{bmatrix}$$

(3,3)

$$\begin{bmatrix} 0 & -\ell_z & \ell_y & f_x & -p_x \\ \ell_z & 0 & -\ell_x & f_y & -p_y \\ -\ell_y & \ell_x & 0 & f_z & -p_z \\ -f_x & -f_y & -f_z & 0 & -E \end{bmatrix}$$

(4,4)

$$\begin{bmatrix} 0 & -\ell_z & \ell_y & f_x & -p_x \\ \ell_z & 0 & -\ell_x & f_y & -p_y \\ -\ell_y & \ell_x & 0 & f_z & -p_z \\ -f_x & -f_y & -f_z & 0 & -E \\ p_x & p_y & p_z & E & 0 \end{bmatrix}$$

(5,5)

Format	Number of components
(2,2)	1
(3,3)	3
(4,4)	6
(5,5)	10



I can decompose this antisymmetric matrix  $J$  of format (5,5) in an antisymmetric matrix  $M$  of format (4,4) and a FOUR-VECTOR  $p$ , with four components. And I will be able to write this in a more compact manner. Quite simply, this will allow me to show the calculation of the action of the Poincaré group on this moment-matrix  $J$  in a more convenient way.

$$\begin{aligned}
 J &= \begin{bmatrix} 0 & -l_3 & l_2 & f_x & -p_x \\ l_3 & 0 & -l_2 & f_y & -p_y \\ -l_2 & l_3 & 0 & f_z & -p_z \\ -f_x & -f_y & -f_z & 0 & -E \\ p_x & p_y & p_z & E & 0 \end{bmatrix} \xrightarrow{\quad} \begin{array}{c} \begin{bmatrix} 0 & -l_3 & l_2 & f_x \\ l_3 & 0 & -l_2 & f_y \\ -l_2 & l_3 & 0 & f_z \\ -f_x & -f_y & -f_z & 0 \end{bmatrix} \quad \begin{bmatrix} -p_x \\ -p_y \\ -p_z \\ -E \end{bmatrix} \\ \begin{bmatrix} p_x & p_y & p_z & E \end{bmatrix} \quad \begin{bmatrix} 0 \end{bmatrix} \end{array} \\
 M &= \begin{bmatrix} 0 & -l_3 & l_2 & f_x \\ l_3 & 0 & -l_2 & f_y \\ -l_2 & l_3 & 0 & f_z \\ -f_x & -f_y & -f_z & 0 \end{bmatrix} \\
 P &= \begin{bmatrix} p_x \\ p_y \\ p_z \\ E \end{bmatrix} \\
 {}^t P &= \begin{bmatrix} p_x & p_y & p_z & E \end{bmatrix}
 \end{aligned}$$

$$J = \begin{pmatrix} M & -P \\ {}^t P & 0 \end{pmatrix} \quad g = \begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix}$$

from that point of view,  
this decomposition is logical



we just have to show the details of  $J' = g \times J \times {}^t g$

$${}^t g = \begin{pmatrix} {}^t L & 0 \\ {}^t C & 1 \end{pmatrix} \quad J' = \begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} M & -P \\ {}^t P & 0 \end{pmatrix} \times \begin{pmatrix} {}^t L & 0 \\ {}^t C & 1 \end{pmatrix}$$

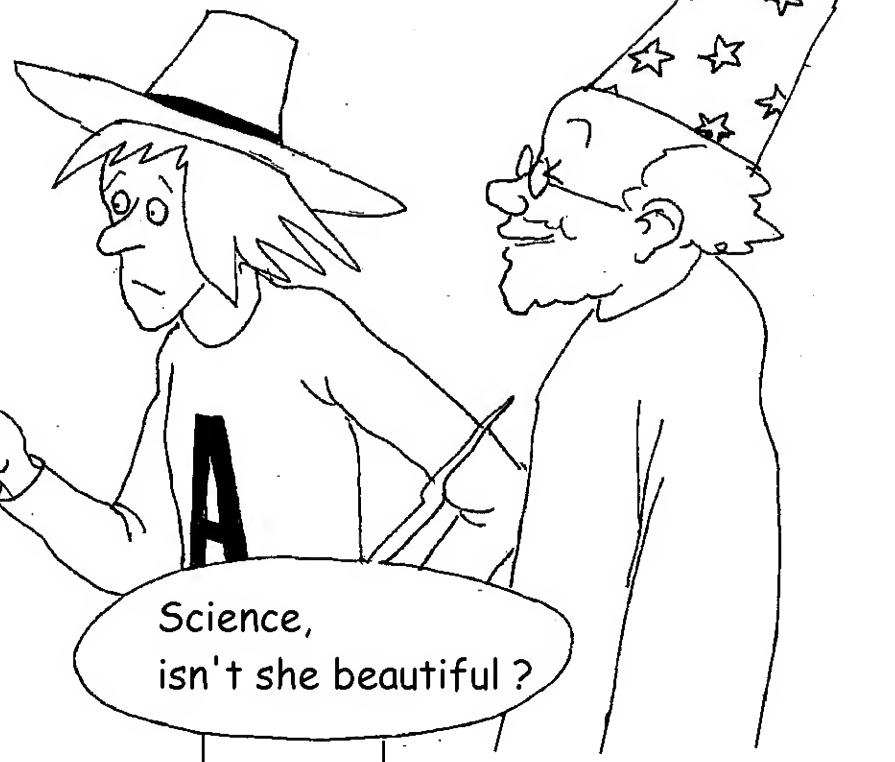
$$J' = \begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} M {}^t L - P {}^t C & -P \\ {}^t P {}^t L & 0 \end{pmatrix} = \begin{pmatrix} LM {}^t L - LP {}^t C + C {}^t P {}^t L & -LP \\ {}^t P {}^t L & 0 \end{pmatrix}$$

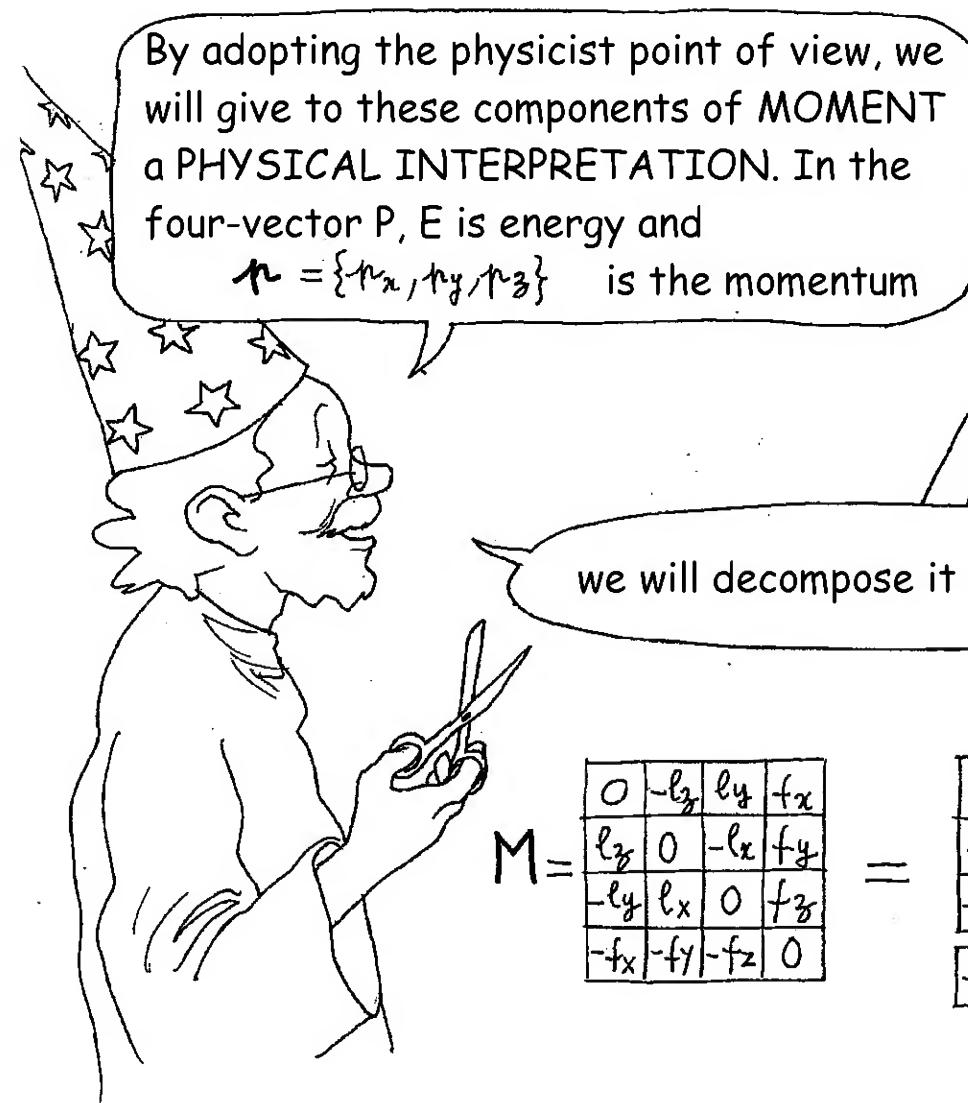
$$M' = LM {}^t L - LP {}^t C + C {}^t P {}^t L$$

$$P' = LP$$

cool stuff. But will these magnificent formulas be of any use to me ?

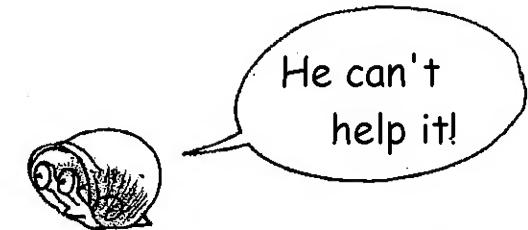
Science,  
isn't she beautiful ?





$$M = \begin{bmatrix} 0 & -l_z & l_y & f_x \\ l_z & 0 & -l_x & f_y \\ -l_y & l_x & 0 & f_z \\ -f_x & -f_y & -f_z & 0 \end{bmatrix} = \begin{bmatrix} 0 & -l_z & l_y & f_x \\ l_z & 0 & -l_x & f_y \\ -l_y & l_x & 0 & f_z \\ -f_x & -f_y & -f_z & 0 \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \\ 0 \end{bmatrix}$$

but this antisymmetric matrix  $M$ , what is that supposed to be ?



$$S = \begin{bmatrix} 0 & -l_z & l_y \\ l_z & 0 & -l_x \\ -l_y & l_x & 0 \end{bmatrix} \quad f = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

$$M = \begin{Bmatrix} S & f \\ \frac{t}{c} & 0 \end{Bmatrix}$$

The velocity  $V$  is implicitly present in the  $L$  matrix of the Lorentz group. If we consider a movement which takes place along a specific direction, for example  $oz$  with a velocity  $V$  and a translation  $\Delta z = c$  and if  $c = V \Delta t$  then we are in a system of coordinates where we follow the particle's movement along this spacetime translation. We then show that the vector  $f$  is null.

The matrix  $S$  is then written:

0	-s	0
s	0	0
0	0	0

Souriau demonstrated in 1972(\*) the  
PURELY GEOMETRIC character of SPIN:  
an antisymmetric matrix (3,3)



The GEOMETRIC QUANTIFICATION method that he invented allows to show that this spin  $S$  can only be a multiple of a fixed quantity:  $\hbar$ . We have seen that the fact that a particle has an electric charge was equivalent of saying that it moves in a space having a FIFTH DIMENSION, the dimension of KALUZA. It's the fact that this dimension is closed onto itself that causes the electric charge to be quantized. In spacetime, there exists a "form of closure" that cause an object to become identical to itself under the action of a  $360^\circ$  rotation. The quantization of Spin, in a certain measure, comes from that property. There exists a close relationship between quantization and closure of a dimension. By exploiting the "group" tool and closure of the 5th dimension, Souriau shows the emergence the Klein-Gordon equation of the Poincaré group (and the Schrödinger equation of the Galilean group, dynamic group managing movement of the non-relativistic material point)

# INVERSION OF ENERGY FOLLOWS FROM INVERSION OF TIME

We've seen earlier that the element from Lorentz group could be written in the form:

$$L = \mu L_0 \quad M = \pm 1$$

where  $L_0$  represents the element of the orthochron subgroup (with does not invert time). In this form the action is written:

$$M' = L_0 M {}^t L_0 - \mu L_0 P {}^t C + \mu C {}^t P L_0$$

$$P' = \mu L_0 P$$

Let's consider the simplest action possible where there is time inversion ( $\mu = -1$ ). In the orthochron  $L_0$ , let's choose the identity matrix  $I$ . Let's cancel the spacetime translation  $C$ . The element of the group is written:

$$g = \begin{pmatrix} -I & 0 \\ 0 & 1 \end{pmatrix}$$

The action on spacetime, the space of trajectories is reduced to:

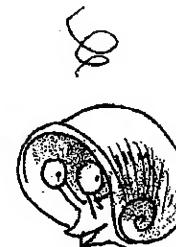
$$\xi' = -\xi \Rightarrow t \Rightarrow -t$$

It's the inversion of the direction of time along the trajectory. The action on the moment is:

$$M' = M \Rightarrow \text{the spin } S \text{ remains unchanged.}$$

$$P' = -P : E \rightarrow -E$$

That's it! It has been hard but we got there!



# APPENDIX 4: THE ANTIMATTER

On page 40 we evoked the idea that for a relativistic material point to have an electric charge  $e$ , we must consider its displacement not in a four dimensional space, but in a space of five dimensions:

$$\{t, x, y, z, \zeta\}$$

$\zeta$  being the fifth dimension, or KALUZA'S DIMENSION.

We had introduced MINKOWSKI'S METRIC on page 137

$$ds^2 = \gamma^t \gamma^x G \gamma^y \gamma^z = dt^2 - dx^2 - dy^2 - dz^2$$

we will start from a KALUZA SPACE, hyperbolic Riemannian, defined by its signature (+ - - -) and its Gram matrix :

$$\Gamma = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} G & 0 \\ 0 & -1 \end{pmatrix} \text{ where } G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The metric of the Kaluza space is :

$$d\Sigma^2 = dt^2 - dx^2 - dy^2 - dz^2 - ds^2$$

$$\tau = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \xi = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} t \\ \tau \end{pmatrix} \quad \Omega = \begin{pmatrix} t \\ x \\ y \\ z \\ s \end{pmatrix} = \begin{pmatrix} t \\ \eta \\ s \end{pmatrix} = \begin{pmatrix} t \\ r \\ s \end{pmatrix}$$

$$d\Sigma^2 = {}^t d\Omega \Gamma d\Omega$$

if we look for the isometry group of this Kaluza space we will find a group which matrix representation looks very much like the one from a Poincaré group but with an extra dimension:

$$\begin{pmatrix} \Lambda & C \\ 0 & 1 \end{pmatrix} \quad \text{with} \quad {}^t \Lambda \Gamma \Lambda = \Gamma$$

this group acts on the points in the Kaluza space:

$$\begin{pmatrix} \Lambda & C \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} \Omega \\ 1 \end{pmatrix} = \begin{pmatrix} \Lambda \Omega + C \\ 1 \end{pmatrix}$$

The vector  $C$  represents this time a translation with five dimensions:

$$C = \begin{bmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \\ \Delta \zeta \end{bmatrix}$$

the translations along dimension  $\zeta$  represents a subgroup of this group:

of which the matrix representation is :

subgroup with 1 parameter

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta \zeta \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} t \\ x \\ y \\ z \\ \zeta \\ 1 \end{bmatrix} = \begin{bmatrix} t \\ x \\ y \\ z \\ \zeta + \Delta \zeta \\ 1 \end{bmatrix}$$

Now Noether's theorem says that a new scalar will be invariant under the action of this subgroup, and this scalar is

THE ELECTRIC CHARGE  $e$

the Kaluza group is constructed from a group  $\Lambda$

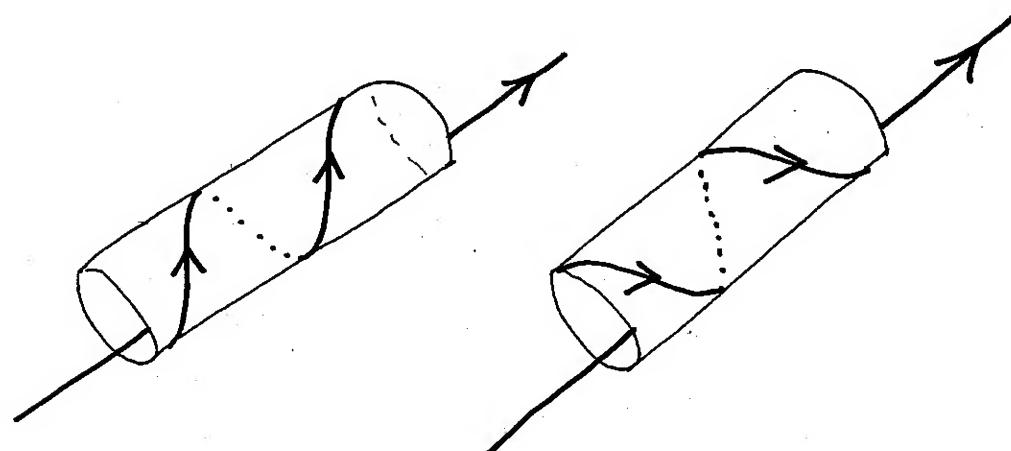
the Lorentz group is one of its subgroups:

$$\begin{pmatrix} L & 0 \\ 0 & 1 \end{pmatrix}$$

here's another subgroup from the Kaluza group

$$\begin{pmatrix} L & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \xi \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} L\xi \\ \mu\xi \\ 1 \end{pmatrix} \text{ with } \mu = \pm 1$$

the elements ( $\mu = -1$ ) of this group invert the fifth dimension. We reuse the sketch from page 42 :  
(the fifth dimension is closed)



The "wrapping direction" of the movement of the particle is reversed. We show (...) that this involves the inversion of the electrical charge  $e$

This cannot represent a geometrical definition of antimatter. A particle has QUANTUM CHARGES and the electric charge  $e$  is only one of them. But we can see the idea coming up: "the antimatter statute depends of a type of movement in a space of higher dimension"

# ORTHOCHRON and ANTICHRON LORENTZ SUBGROUP

The LORENTZ GROUP  $L$  has four components

$L_n$  (neutral),  $L_s$  (inverts space),  $L_t$  (inverts time),  $L_{st}$  (inverts space and time)

The "neutral component" is a subgroup which contains the unit element, unlike the three other sets and does not inverts neither time or space. Below, a few matrices which belongs to the sets ( $\in$  means "belongs to" and  $\{ \}$  means set)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in \{L_n\}; \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \in \{L_s\}; \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in \{L_t\}; \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \in \{L_{st}\}$$

# APPENDIX 5

## TWIN GROUP

We can regroup these four sets of matrices in two subsets :

$$L_0 \text{ (orthochron)} = \{L_n, L_s\} \quad L_a = \{L_t, L_{st}\}$$

The first subset is a subgroup of the Lorentz group. This regrouping allows us to write :

$$L = \mu L_0 \text{ with } \mu = \pm 1 \text{ because } L_t = -L_s \quad ; \quad L_{st} = -L_n$$

In this large matrix calculation that we didn't dare putting on these pages (but that you could easily follow), the most general "ACTION" of the components of the Poincaré group on its "moments space" contains the relation (Souriau 1972)



$$\begin{pmatrix} E' \\ p'_x \\ p'_y \\ p'_z \end{pmatrix} = L \times \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix} = \mu L_0 \times \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

The elements  $\mu = -1$  correspond to the ANTICHRON transformations which invert time. The identity matrix (4,4)  $I$  is part of the Lorentz group. When we limit ourselves to just inverting time, we see that it inverts the energy, but also the momentum  $p$

$$\mathbf{p} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

$$E' = -E \quad \mathbf{p}' = -\mathbf{p}$$

If we take the Kaluza group

$$\begin{pmatrix} \Lambda & C \\ 0 & 1 \end{pmatrix}$$

all calculations can be redone in 5D and we will obtain in particular with:

$$\pi = \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \\ e \end{pmatrix} \quad \pi' = \Lambda \pi$$

We can decompose the group  $\Lambda$  in two components, one is orthochron and the other antichron, and write

$$\Lambda = \mu \Lambda_0 \quad \text{with} \quad \mu = \pm 1$$

the ANTICHRONS components ( $\mu = -1$ ) invert

- The energy  $E$
- The momentum  $p$
- The electrical charge  $e$

We can express  $\Lambda$  by using the orthochron subset  $L_0$  of the Lorentz group and, by adding ( $\lambda = \pm 1$ ) we introduce (in the two sheets) matter-antimatter duality

$$\Lambda = \begin{bmatrix} \mu L_0 & 0 \\ 0 & \lambda \end{bmatrix}$$

The subgroup from the Kaluza group we have chosen is then written

$$\begin{bmatrix} \mu L & 0 & \Delta \xi \\ 0 & \lambda & \Delta \xi \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \xi \\ \xi \\ 1 \end{bmatrix}$$

# APPENDIX 6: IMAGINARY SPACES DO YOU HAVE A SOLE ?

We remember that by interacting two cosmical subsets of opposite masses and energies, we represented these two sheets like the covering of a projective, which in the case of two dimensions  $(t, x)$  became a BOY SURFACE (\*)

We also envisioned that the two "poles", one representing the BIG BANG and the other the BIG CRUNCH, instead of being identified, corresponded to a gateway, a bridge linking the two sheets. This made the singularity disappear and moreover, in 2D, gave to the universe-object the topology of a torus  $T^2$  arranged in a covering of two sheets of a Klein  $K^2$  bottle (more readable in "Topo the world"). The frontier space is then a circle  $S^1$

(\*) described in details in "Topo the world"

If we place ourselves in 5D we must suppose that we can construct a solution with two metrics of the type

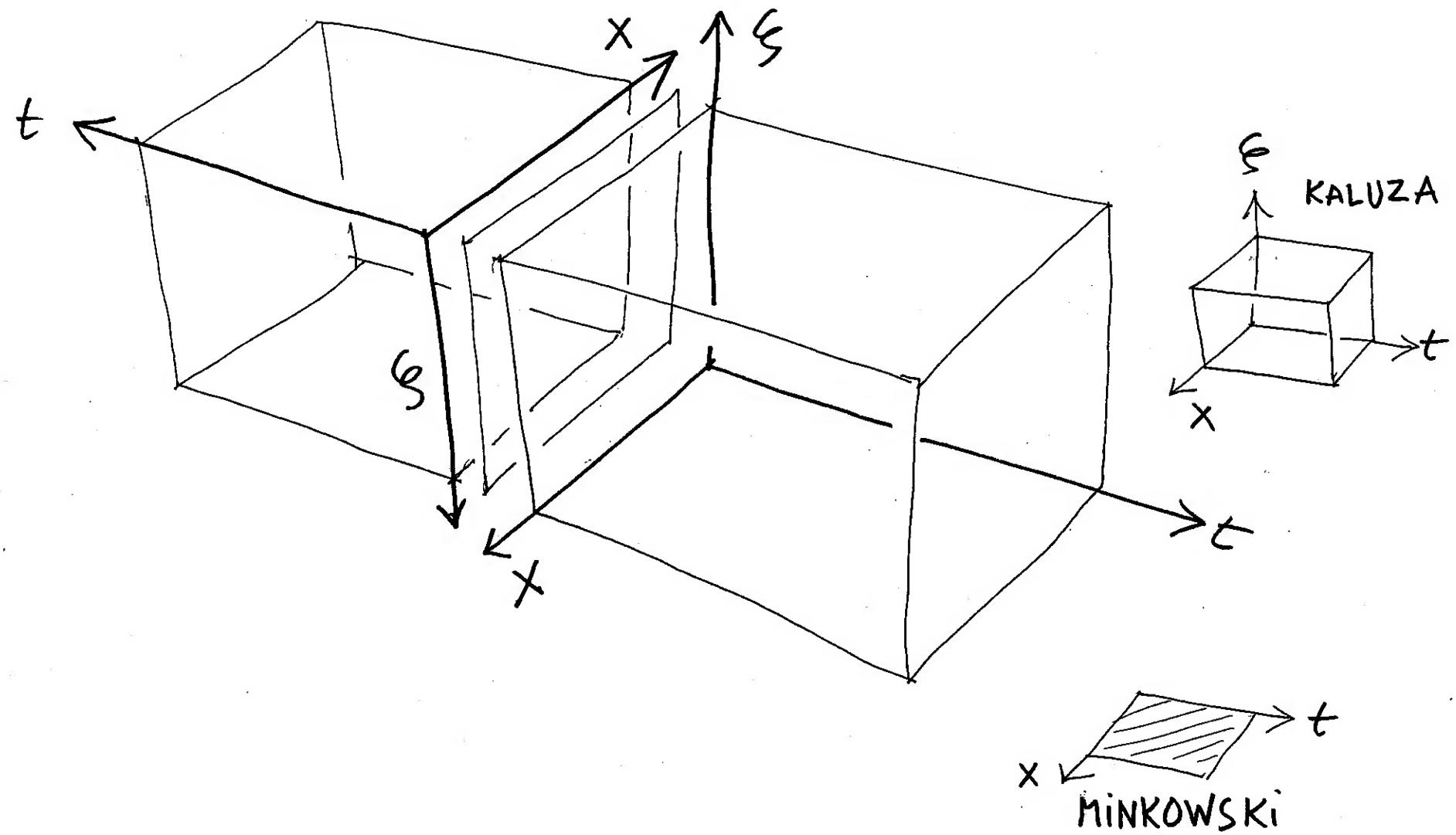
$$d\sum^2 = R^2 [dt^2 - dx^2 - dy^2 - dz^2 - d\zeta^2]$$

In the primitive Universe (see FASTER THAN LIGHT), before the BREAKING OF SYMMETRY the two scale factors (Warp factors) are supposedly equal. At the juncture, there is a dimensional degeneracy. The metric of the frontier-space then becomes :

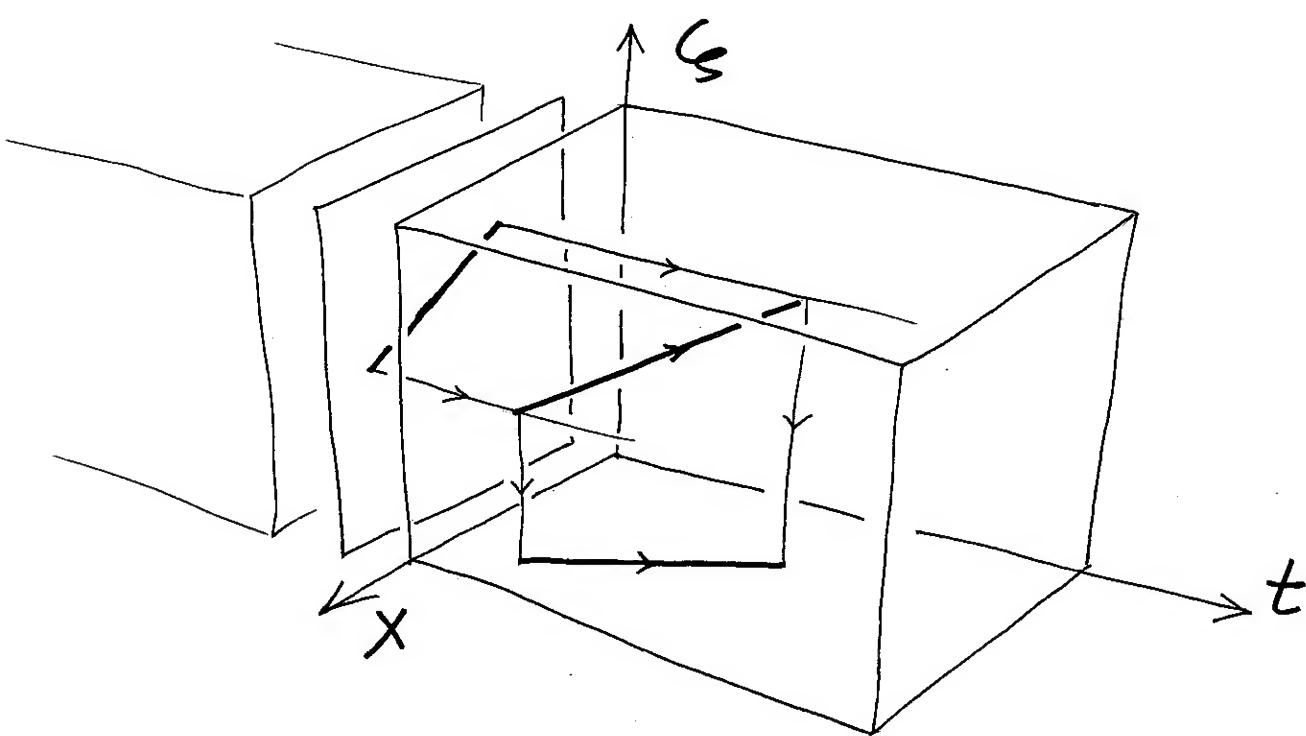
$$d\sigma^2 = R_{\min}^2 [-dx^2 - dy^2 - dz^2 - d\zeta^2] < 0$$

IN THIS FRONTIER-SPACE, THE LENGTH IS PURE IMAGINARY  
CAN IT BE ASSIMILATED TO PURE IMAGINARY TIME ?

IN ANY CASE, WHAT (META)PHYSICAL SIGNIFICANCE SHOULD WE GIVE TO THIS  
GEOMETRICAL STRUCTURE ?



the "TOY MODEL"



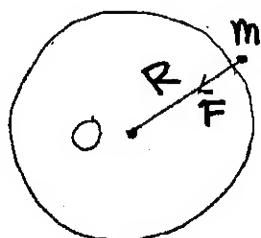
Nobody ever ventured to give some model of what could be the CONSCIENCE with its corollary : CHOICE. Above we have an amusing image where a "line of destiny", achrone, inscribed in this frontier space  $(x,y,z,\zeta)$  of signature  $(- - - -)$  can project itself in an infinity of possible ways in one of the two sheets of spacetime  $(X,t)$ , the choice of such or such a projection representing a DEGREE OF FREEDOM

here, we stop...



# APPENDIX 7: NEWTONIAN SOLUTIONS

In 1934, Milne and Mac Crea created a big surprise when, by just using Newton's law and a little bit of calculations, they emerged Friedman's equation, the law of evolution of the characteristic dimension  $R$  of the universe. The method consists by considering a small part of the universe, contained in a sphere of radius  $R$  centered on  $O$ ,  $\rho$  being the matter density in this sphere.



Then we look what is the acceleration  $R''$  to which this mass is submitted by supposing that the point  $O$  is fixed. Then we can show that the radial force to which this mass  $m$  is submitted is limited to a mass  $M = \frac{4}{3}\pi R^3 \rho$  which would be situated in  $O$  and which represents the mass contained in this sphere of radius  $R$ .

$$F = -\frac{G m}{R^2} \frac{4}{3}\pi R^3 \rho = m R''$$

we obtain the differential equation :

$$R'' = -\frac{1}{R^2} \left( \frac{4\pi G \rho R^3}{3} \right)$$

If mass is conserved  $\rho R^3 = C^{\text{te}}$  We obtain Friedman's equation

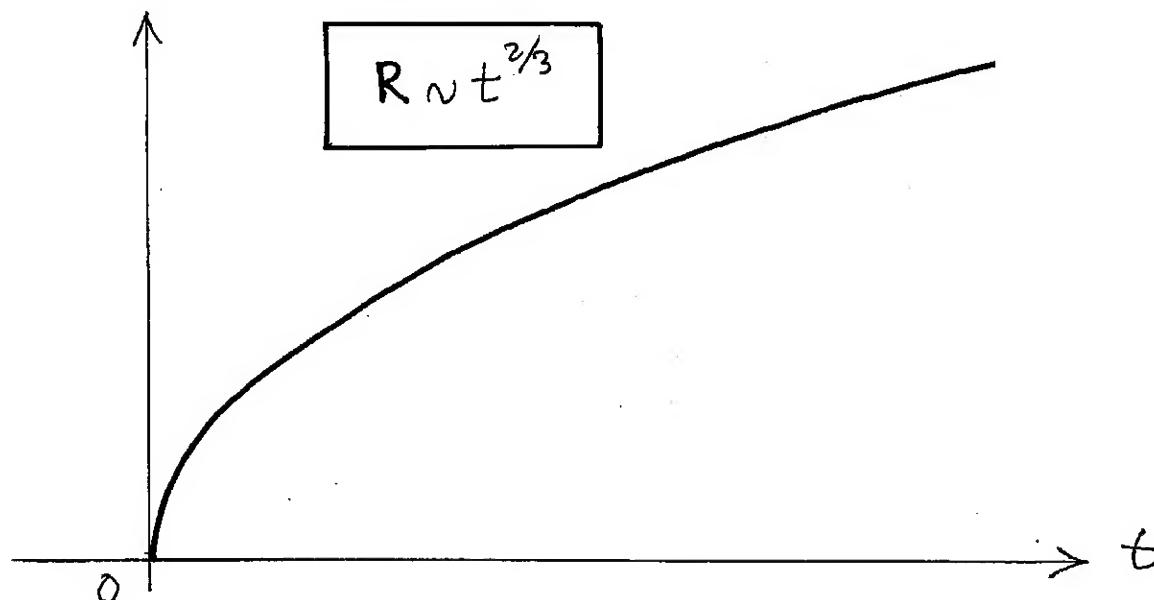
$$R'' = -\frac{a^2}{R^2}$$

which has three types of solutions which all show a deceleration, infinite for  $R = 0$  then decreasing as time increases and  $R(t)$  expands. We'll be looking for the law in

$$R \sim t^n$$

$$R' = n a^2 t^{n-1} ; R'' = n(n-1) a^2 t^{n-2} ; R^2 R'' = n(n-1) a^6 t^{3n-2}$$

which leads to the parabolic solution:



Imagine now that the evolution of the Universe is governed by two type of contents, one being positive masses  $m^+$  and the other being negative masses  $m^-$ . Moreover, like we tried to make you understand in this comic album, this expansion is being played through two SCALE FACTORS  $R^+$  and  $R^-$  (Warp factors)

Let's consider a positive mass  $m^+$  situated on a sphere of radius  $R^+$  whose center is assumed to be fixed. Within a Newtonian approximation let's calculate the acceleration  $R^{+''}$  that this mass undergoes. It can be calculated by considering, like before, the quantity of positive mass contained in this sphere (and brought back at its center  $O$ ):

$$\frac{4}{3}\pi p^+ R^{+3}$$

We must take into account of the APPARENT MASS of the negative mass contained in this sphere which is :

$$\frac{4}{3}\pi p^- R^{+3} \quad \text{avec} \quad \frac{p^-}{p^+} = \frac{R^{+3}}{R^{-3}}$$

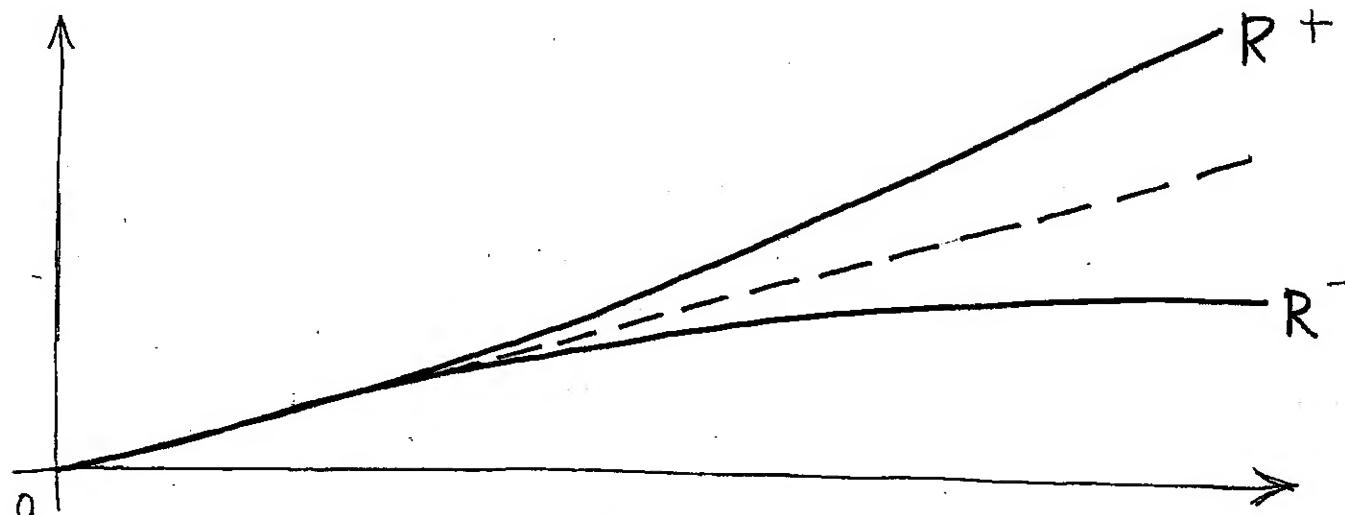
The differential equation giving  $R^+(+)$  is then :

$$R^{+''} = -\frac{Gm^+}{R^{+2}} \times \frac{4\pi}{3} (p^+ - p^-) = -\frac{a^2}{R^{+2}} \left(1 - \frac{R^{+3}}{R^{-3}}\right)$$

By using the same reasoning and using this time the  $R^-''$  acceleration undergone by a mass  $m^-$  and by taking the constant (arbitrary) equal to 1, we will have this system of two coupled differential equations :

$$\left\{ \begin{array}{l} R^{+''} = -\frac{1}{(R^+)^2} \left( 1 - \frac{(R^+)^3}{(R^-)^3} \right) \\ R^{-''} = -\frac{1}{(R^-)^3} \left( 1 - \frac{(R^-)^3}{(R^+)^3} \right) \end{array} \right.$$

which allows the linear solution (unstable)  $R^+ = R^- \sim t$



The instability of the solution, by supposing that the positive masses undergo a late acceleration will give the illusion of the action of a DARK ENERGY.

These two worlds composed of energies and masses of opposite signs do interact. In the case showed in the previous page, the denser negative masses accelerate the phenomenon of expansion of the positive masses, associated to the scale factor  $R^+(+)$ . The opposite phenomenon happens in the "negaworld" where observers, composed of negative masses themselves, and receiving signals transported by NEGATIVE ENERGY PHOTONS, would note a deceleration of the expansion phenomenon.

The start of the curve, where expansion seems linear, could seem incompatible with observations. But at this point intervenes a SYMMETRY BREAKDOWN and a VARIATION OF THE CONSTANTS, in particular of the speed of light. Without it the widespread homogeneity of the primitive universe is not explicable. All of this has been discussed in this album:

**FASTER THAN LIGHT**